A comparison between tomographic and geodynamic models of the Earth's mantle

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We compute tomographic images of the short and long wavelength expansions of a 3-D spherical convection simulation to a resolution comparable to that of a recent seismic model of the whole mantle. The simulation incorporates realistic thermal effects associated with convection in the mantle, namely, 20% bottom heating and phase transitions at 410 km and 670 km. A depth-dependent viscosity profile, and the history of plate motions starting in the mid-Mesozoic are also imposed. We show that tomography recovers the image of the convection simulation well, making it a useful tool to investigate physical parameters characteristic of mantle convection, in conjunction with geodynamic modeling. Our results indicate that the part of the temperature field corresponding to cold mantle heterogeneity in the convection simulation agrees relatively well with the fast velocities derived from tomography, but that current convection computations are lacking some key ingredients necessary to reproduce the character of upwellings correctly. Contingent upon the accuracy of the time dependence of the plate reconstruction, we place an upper bound of -4 $MPa.K^{-1}$ on the Clapeyron slope of the phase transition at 670 km.

Comparisons between the images of the short and long wavelength model (developed up to degrees 32 and 12, respectively) provide insight into the spectral leakage in inversions of actual waveform data. We show that contamination of long-wavelength models by small-scale structure is present at all depths, increases with spherical harmonic degree, and has a peak magnitude of the order of 1% perturbation from *PREM*. Although this effect is small in absolute terms, it is significant on the scale of the models and casts doubts on the reliability of the smallest amplitude component of the heterogeneity pattern in long wavelength seismic images of the whole mantle.

1. INTRODUCTION

Three-dimensional images of the Earth's deep mantle obtained by seismic tomography provide us with increasingly accurate constraints on the large scale dynamics that drive plate motions. Since the earliest global tomographic models of the upper mantle [*Woodhouse & Dziewonski*, 1984; *Nataf et al.*, 1986], and even earlier regionalized models (e.g. [*Lévêque*, 1980; *Okal*, 1983]), it has been shown that the velocity structure in the uppermost mantle correlates with surface tectonics and that this correlation breaks down below about 250 km, with, in particular, a strong "degree 2" pattern in the transition zone [*Masters et al.*, 1982; *Souriau & Souriau*, 1983]. This pattern is shifted to the west with respect to the shallower signature at degree 2 of plate tectonics, and is correlated with a degree 2 pattern in seismic attenuation [*Romanowicz et al.*, 1987; *Romanowicz*, 1990; *Montagner & Romanowicz*, 1993]. In the lowermost mantle, a large scale degree 2 pattern of geodynamical significance, correlated with that of the geoid, has also been found in the earliest studies [*Dziewonski et al.*, 1977; *Hager et al.*, 1985].

Over the years, as resolution in tomographic models has improved, greater details concerning the large scale flow in the mantle have been obtained. In particular, recent models with high resolution around subduction zones have allowed seismologists to image subducted lithospheric slabs through the upper mantle transition zone and into the lower mantle, and have shown a variety of situations, with slabs appearing to be lying flat at the upper-mantle/lower-mantle transition in some regions, and penetrating to large depths in the lower mantle in others. This has been shown both in regional studies (e.g. [*Spakman et al.*, 1989; *Zhou & Clayton*, 1990; *van der Hilst et al.*, 1991; *Fukao et al*, 1992; *van der Hilst & Seno*, 1993; *van der Hilst*, 1995]), and in whole mantle models (e.g. [*van der Hilst et al.*, 1997; *Grand*, 1994]). Because of a less favorable geometry in mid-oceanic areas, regions of upwellings, as expressed by low velocities in tomographic regions, are less well resolved, and, in particular, this has given rise to a vigorous debate regarding the continuity at depth of low velocities associated with mid-ocean ridges ([*Zhang & Tanimoto*, 1992; *Su et al.*, 1992]).

While high resolution images increasingly will become available on a regional scale, global models of the whole mantle in which the structure is expressed as a truncated spherical harmonic expansion in the horizontal dimension provide useful constraints on the long wavelength characteristics of mantle circulation and how it relates to surface tectonics. Until recently, such models were limited to degree 12 (e.g. [*Su et al.*, 1994; *Li & Romanowicz*, 1996]), and are now progressively reaching higher degrees [*Masters et al.*, 1996; *Mégnin & Romanowicz*, 1997; *Liu & Dziewonski*, 1998; *Mégnin & Romanowicz*, 1999b], with the newest models showing good agreement in their low degrees (up to degree 8-12) with the previous generation. A common feature of these models is their prominent long wavelength component (*e.g.* [*Su & Dziewonski*, 1991]), with peaks at degree 2 in the transition zone and lowermost mantle, and predominance of degrees 1-8 in the upper mantle.

These features provide strong constraints for numerical modeling of convection in the mantle, in that a variety of ingredients, such as level of heating from the bottom of the mantle, nature of the 670 km discontinuity, viscosity stratification, and boundary conditions at the top of the mantle, contribute to the spectral characteristics of the synthetic velocity models derived from numerical computations, which ultimately need to match the seismological results.

While good agreement between mantle models resulting from convection calculations and seismic tomography would go a long way towards improving our understanding of mantle dynamics, geodynamicists and seismologists have traditionally failed to produce compatible images, particularly the characteristic length scale of heterogeneity. Simple fluid dynamic models predict dominant wavelengths of the order of mantle depth [*Busse & Whitehead*, 1971]. This is in contradiction with both geoid observations (e.g. [*Lerch et al.*, 1982]) and images derived from seismic tomography (e.g. [*Tanimoto*, 1990; *Su et al.*, 1994; *Li & Romanowicz*, 1996; *Grand et al.*, 1997]) which find, as mentioned above, that the spectrum of heterogeneity in the mantle peaks at scales of 10,000 to 20,000 km. The disagreement remains with spherical axisymmetric convection simulations which predict a pattern of heterogeneity with a much broader spectrum (e.g. [*Machetel*, 1990]). The discrepancy was attributed to the presence of a filtering effect in the tomography and to computational limitations that did not permit the imaging of very fine scale mantle structure.
More recently, it was shown that, by introducing a depth dependent viscosity profile in the convection simulations [*Bunge et al.*, 1996], and more importantly by imposing plate motions [*Davies*, 1988; *Bunge & Richards*, 1996], a convective planform could be generated, characterized by the predominance of long wavelength circulation, which considerably improved the spectral agreement between geodynamic calculations and seismological models.
Although differences between seismic images of the mantle and convection calculations remain, the similarities now allow direct comparisons. In particular, the thermal effect of phase transitions on the convection planform in a realistic geometry produces spectrally localized features at transition zone depths. It is possible to use the convection models in conjunction with a realistic distribution of seismic data to estimate whether such features, if they exist, would be recovered in the inversion of real seismic data.

The spatial resolution of tomographic models and models resulting from convection calculations, however, are very different. Resolution in geodynamic models is controlled by the numerical mesh, (in the case of the convection model in this study, 10 million finite elements were used, corresponding to heterogeneity of the order of 50 km). In contrast, the resolution achieved by seismological models is limited by the filtering that the uneven data sampling, the regularization and the parametrization introduce in the inversion. For the purpose of comparison between seismological and numerical modelling, the recent "high resolution" models, as mentioned earlier, are accurate mostly in regions around subduction zones, so that long wavelength, global models are better suited for such comparisons. However, contamination of long wavelength seismic images by unmodeled small scale heterogeneity may result in a distorted model. Therefore, predicted seismic images of convection models must be computed by applying the same filtering process as has been applied to the real earth in the inversion of actual data. Such an approach allows to assess how a particular convection model would be "seen" through the blurred glass of global seismic tomography, and therefore to perform meaningful comparisons.

The issue of contamination of long wavelength tomographic models by small scale heterogeneity is central to the interpretation of seismic images in terms of mantle dynamics. By analyzing frequency shifts in the normal mode ${}_0S_{40}$, [*Snieder et al.*, 1991] have suggested that the presence of fine scale subduction zones and spreading ridge structures introduces appreciable spectral leakage into the even-degree expansion of the local frequency perturbation, up to spherical harmonic degree 6. However, the local frequency perturbations only represent the effects of the radially integrated earth structure. In the present study, we analyze how aliasing impacts the velocity models in different depth ranges using synthetic computations based on the combination of a recent numerical convection model [*Bunge et al.*, 1998] and the seismic sampling provided by a recent global tomographic study [*Li & Romanowicz*, 1996].

2. METHOD

The resolution of seismic models is generally tested by constructing synthetic input patterns such as 'checkerboards' and evaluating their recovery. In regional and global studies, more realistic synthetic structures, such as slabs, are also commonly evaluated (e.g. [*Spakman et al.*, 1989; *Fukao et al*, 1992; *van der Hilst et al.*, 1997]). [*Pulliam & Stark*, 1994] have also assessed the overall reliablility of images from travel time tomography. Realistic numerical convection models provide the advantage of allowing to assess the resolution by seismic tomography of both large scale global features and localized heterogeneity, specific for example to subduction zones or hot upwellings, and to determine whether physical parameters such as the strength of phase transitions can be estimated.

Several studies have used tomographic sampling of convection simulations as an effective way to explore similarities between geodynamic and tomographic mantle models. [*Johnson et al.*, 1993] used the global distribution of available $SS-S$ and $ScS-S$ travel times to image spherical convection simulations of [*Tackley et al.*, 1994] and found general agreement between the spectra of the convection simulations and of the seismic models derived from these synthetic travel time data. [*Mégnin et al.*, 1997] argued that waveform tomography can successfully reproduce the spectral signature of various convection scenarii, each characteristic of a particular aspect of the physics of mantle convection. Their method consisted in applying the resolution matrix derived in the computation of the tomographic model *SAW12D* [*Li & Romanowicz*, 1996] to image eight convection calculations by [*Bunge et al.*, 1997]. Although convenient, this approach involves strong linear approximations and is not suitable for the analysis of short-wavelength contamination. An analysis of the aliasing of degrees 7-12 into a degree 6 model was attempted but the results proved strongly dependent on the choice of roughness penalty used in the modeling. It is important to note that such studies of convection calculations have so far focused on particular physical parameters corresponding to a dynamical mantle possibly very different from the actual earth. A new generation of 3-D spherical convection simulations of [*Bunge et al.*, 1998] that includes the principal thermal effects expected to be present in the mantle allows to make somewhat more meaningful comparisons between geodynamic and tomographic models, even though the currently available convection simulations only represent one step closer to a more realistic dynamic Earth. In the future, accurate modeling of mantle processes will require accounting for effects on the convective planform such as chemical and rheological stratification, partial melting, and temperature-dependent viscosity.

In this study, we consider the source-receiver distribution used in computating the whole mantle model *SAW12D*, consisting of 9,000 SH body wave seismograms low-pass filtered at 32s and 7,500 first and second orbit Love wavetrains, low-pass filtered at 80s. The body wave seismograms correspond to a collection of over 28,000 body phases, corresponding to S, SS, S_3, S_4, ScS, ScS_2, ScS_3, S_{diff} and the corresponding depth phases (sS, etc...). They are catalogued in Table 1. Modeling of waveforms with accurate body wave kernels [*Li & Tanimoto*, 1993; *Li & Romanowicz*, 1995] gives access to information contained in many phases, including those that arrive close together in time, thus providing improved sampling compared to single phase travel time delays.

We compute synthetic waveforms corresponding to the actual source-station configuration, in a predefined geodynamic model and invert them for up to spherical harmonic degree 12. The synthetic waveform dataset and its subsequent inversion are computed using the 'nonlinear asymptotic coupling theory' (*NACT*) developed by [*Li & Romanowicz*, 1995]. This formalism takes into account the coupling of multiplets along and across mode branches, a prerequisite for the correct modeling of body waveforms [*Li & Tanimoto*, 1993]. Because of the zeroth order asymptotic expansion of the coupling terms, the sensitivity of the waveforms is restricted to the two-dimensional structure along the great-circle path [*Romanowicz*, 1987]. Resolution in the lowermost mantle is improved by a data selection scheme that isolates the different energy wavepackets of a given seismic trace [*Li & Romanowicz*, 1996]. This enhances the low amplitude phases, in particular the S diffracted along the core-mantle boundary (S_{diff}) which constitutes the primary source of information on lowermost mantle heterogeneity [*Mégnin & Romanowicz*, 1999a].
All the models in this study are parametrized radially in Legendre polynomials up to degree 5 in the upper mantle and 7 in the lower mantle, and laterally in spherical harmonics, up to degree 12 for *SAW12D*, and up to degree 32 for the convection model.

2.1. Temperature to velocity scaling

In the absence of chemical heterogeneity, the shear velocity perturbation $\delta \ln V_s$ is obtained from the temperature field δT from *GEMLAB 1* through the relation:

$$\delta \ln V_s \,=\, \frac{\alpha}{2}\,(1\,-\,g)\,\delta T \qquad (1)$$

where α is the thermal expansivity $-\frac{\partial \ln \rho}{\partial T}|_P$, g is the second Grüneisen parameter $\frac{\partial \ln \mu}{\partial \ln \rho}|_P$, μ the shear modulus, and P the pressure field.

In this study, we use as reference the thermal expansivity profiles of the spherically symmetric model *PREM* [*Dziewonski & Anderson*, 1981]. The parameter g is poorly constrained and we assign it the constant value of 4 throughout the mantle, which is within the range of estimates of [*Anderson, 1989*, p. 120]. Equation 1 thus reduces to

$$\delta \ln V_s(r,\theta,\phi) = \frac{\alpha(r)}{2}(1 - g)\,\delta T(r,\theta,\phi) \qquad (2)$$

2.2. Aliasing of small-scale structure into long wavelength models

We compute a set of seismograms for the input model truncated at degree 12 and invert for shear velocity perturbations from *PREM* up to degree 12. The resulting model is 'unaliased' in the sense that it is not contaminated by small scale heterogeneity (*i.e.* by structure at shorter wavelengths than degree 12). The difference between the unaliased and the input models results from the combination of several effects: 1) Gaps in spatial coverage; 2) Varying degree of sensitivity of the waveforms to the wavelength of structure (for instance very long period surface waves aren't sensitive to very small scale heterogeneity); 3) Inappropriate choice of tuning parameters and regularization in the inversion; 4) The parametrization, and, in particular, 5) possible artefacts due to separate vertical parametrization of the upper and lower mantle. In the present study, the same theory is used in the forward and the inverse problem. This analysis therefore cannot provide insight into the distortion caused by approximations in the formalism. The effect of modeling the waveforms with an asymptotic formalism is discussed in [*Clévédé et al.*, 1999] in the context of upper mantle tomography from long-period surface waveforms.

We compute a second set of synthetic seismograms with the same source-receiver distribution as the former, but this time through the short wavelength expansion of the model (degrees 0-32) and invert again for heterogeneity up to degree 12. In this respect, the procedure is analogous to the inversion of actual data as it allows us to examine possible aliasing of degree 13-32 structure into the degree 0-12 model. Hereafter, we refer to that model as the 'aliased' model.

We refer to the difference between the unaliased and the input model as the 'inherent' error in the model. In the absence of error arising from misjudgement in the selection of roughness penalty, the unaliased model is indeed the best possible model that can be derived with our methodology, given the data distribution, its frequency band and the model parametrization.

Although the effect of spectral leakage is clearly present on a given seismogram, its impact on velocity models may be modest because data averaging tends to remove incoherent structure. We estimate the amount of aliasing in tomographic images by subtracting the heterogeneity of the unaliased model from that of the aliased model. The resulting 3-D image then represents the spectral leakage of degrees 13-32 into the aliased model. This measure only represents an approximation of the aliasing in inversions of actual data, since the mantle clearly contains heterogeneity of finer scale than used for the purpose of this study. Although *GEMLAB 1* was developed up to degree 400, we truncated it to degree 32 due to computational limitations. The spectrum of *GEMLAB 1* is dominated by the long wavelengths, allowing us to model the first order effect of the contamination of the image from small-scale heterogeneity.

2.3. Inversion procedure: tuning the regularization

We use the same inversion procedure as that of [*Li & Romanowicz*, 1996]. A detailed description is given in that paper and we present here only the aspects relevant to this study:
The inverse problem is solved by making use of a stochastic formulation [*Jackson*, 1979] and the finite-dimensional model is obtained by the recursive application of the following Newton scheme to the Fréchet derivatives of the L^2-norm objective function [*Tarantola & Valette*, 1982]:

$$\begin{aligned} m_{i+1} &= m_i + \left(\mathbf{A}_i^T \mathbf{C_d}^{-1} \mathbf{A}_i + \mathbf{C_m}^{-1}\right)^{-1} \\ &\times \left(\mathbf{A}_i^T \mathbf{C_d}^{-1} \left(\mathbf{d} - \mathbf{u}(m_i)\right) - \mathbf{C_m}^{-1} m_i\right) \quad (3) \end{aligned}$$

Where $\mathbf{d}$ is the data, $\mathbf{u}(m_i)$ the synthetic seismogram evaluated for model m_i and $\mathbf{A}_i$, the matrix of partial derivatives with respect to the parameters of model m_i. The component of the model covariance matrix $\mathbf{C_m^{het}}$ corresponding to perturbations in lateral heterogeneity $dlnV_s$ is constructed according to:

$$\begin{aligned} \mathbf{m}^T \quad \left(\mathbf{C_m^{het}}\right)^{-1} \mathbf{m} &= \int_{\oplus} \eta_1 m^2 + \eta_2 \left(\frac{\partial m}{\partial r}\right)^2 \\ + \quad \eta_3 \left(\frac{\partial^2 m}{\partial r^2}\right)^2 &+ \eta_4 \mid \nabla_1 m \mid^2 d\oplus \qquad (4) \end{aligned}$$

where $\oplus$ is the volume of the mantle, and ∇_1 the surface gradient operator. A similar expression applies to the perturbations in the topography of mantle discontinuities. The selection of damping parameters η_i is arbitrary. In this study, the problem of determining the optimal set of damping parameters is posed as an inverse problem in which we seek to identify the η_i that will minimize the Euclidian distance between the output model m_{i+1} and the target input geodynamic model. To minimize the number of iterations, we use as starting model m_i the target model truncated to degree 8. The optimal set of model norm, lateral derivative, and first and second radial derivatives is computed with a simulated annealing algorithm coupled with a downhill simplex method [*Press et al.*,1994].

3. CONVECTION MODEL GEMLAB 1

We use as input model the time-dependent 3-D spherical convection calculation 'Geodynamical Model of Los Alamos and Berkeley' (*GEMLAB 1*) derived by [*Bunge et al.*, 1998] and characterized by:

- Imposition of the plate motion history starting in the mid-Mesozoic (119 Ma).
- An exothermic phase transition at 410 km with Clapeyron slope of $2MPa.K^{-1}$.
- An endothermic phase transition at 670 km with Clapeyron slope of $-4MPa.K^{-1}$.
- A rheology whereby the lithosphere and the lower mantle are 40 times more viscous than the upper mantle.
- 20% heating from the core.

This model is arguably the most realistic numerical models available to date, in that it predicts a spectrum of heterogeneity peaking at low degrees. However, we must keep in mind that it is, for the purpose of our study, only a convenient proxy for the real Earth. The strength of the Clapeyron slope of the two discontinuities is still an object of controversy, as is the magnitude of the viscosity stratification between upper and lower mantle. In addition the viscosity contrast imposed between lithosphere and asthenosphere is probably several orders of magnitude larger in the Earth than in *GEMLAB 1*.

3.1. Resolution analysis

Two sets of synthetic seismograms are computed from *GEMLAB 1*: using its degree 32 expansion (Figure 1) and using its degree 12 expansion (Figure 2). The resulting 'data sets' are inverted for heterogeneity up to degree 12, producing an aliased (Figure 3) and unaliased (Figure 4) model, respectively. Although a damping of amplitude is observed, maps at given depths do not display noticeable spatial distortion of the input structure. Spectral heterogeneity maps (*SHM*s) of the models are shown in Figure 5 and 6. The plots represent the rms amplitude of each model as a function of wavelength (horizontal axis) and depth (vertical axis). Horizontal lines are traced at 410, 670 and 2600 km depths. Since amplitudes in the mantle are dominated by the top and bottom boundary layers, little can be seen at other mantle depths. In Figure 6, we normalize the amplitudes of Figure 5 by the rms at each depth. Figures 5 & 6 (a) show the first 12 degrees of the input model *GEMLAB 1* and correspond to Figure 2. The uppermost mantle is characterized by a spectrum with amplitudes at all degrees and dominated by peaks at degrees 1, 4 and 5 which are most prominent between 100 km and 150 km depth. The signature of the phase transitions is observed at degrees 2 to 5, 7 and 10 between 400 km and 700 km depth. The heterogeneity pattern in most of the lower mantle is dominated by the long wavelengths. The D" region displays a broad spectrum, peaking at degrees 2-3 and showing energy up to degree 10.

Figures 5 & 6 (b) are the spectral representation of the aliased model in Figure 3, resulting from the inversion for up to degree 12 of the synthetics traced through the degree 32 model. In the uppermost mantle, the low degree spectral peaks are well recovered although a slight loss of amplitude and radial smearing of the degree 6 heterogeneity is observed. In the bottom of the mantle, the spectral peaks at degrees higher than 6 in the input model are noticeably attenuated.

Figures 5 & 6 (c) correspond to the model of Figure 4 and represent the unaliased model resulting from the inversion for up to degree 12 of the synthetics traced through the long wavelength (degree 12) model. Here again, a modest amount of radial smearing is observed in the amplitude peaks of the upper mantle. The spectral character of the input model is nonetheless well preserved.

Radial smearing in the upper mantle is also apparent in the rms profiles of Figure 5 (d). The radial character of the rapid amplitude excursions associated with the plates and the phase transitions in the input model (solid line) cannot be resolved perfectly with 32s body waves: vertical averaging is present in both the aliased (dotted line) and the unaliased (dashed line) models.

Figures 5 & 6 (e) and 7 represent the inherent error obtained by subtracting the model in Figure 2 (*GEMLAB 1*, expanded up to degree 12) from the model in Figure 4 (its unaliased version). The contamination appears strongest in the uppermost mantle and the transition zone where the strength of the amplitude peaks is underpredicted and the peaks are spread radially. The amplitude of the error is nonetheless modest when compared to the scale of the models: the spurious heterogeneity introduced never exceeds 1%.

In Figure 8 (top), we plotted the degree by degree euclidian distance between the unaliased and the long wavelength input models, relative to the former. The distance for spherical harmonic degree s between two models m and p, expanded radially in polynomials up to degree kmax is defined as:

$$D_s \equiv 100 \times \left(\frac{\sum_{k=0}^{kmax} \sum_{t=0}^{2*s+1} (m_k^{st} - p_k^{st})^2}{\sum_{k=0}^{kmax} \sum_{t=0}^{2*s+1} (m_k^{st})^2} \right)^{\frac{1}{2}} \qquad (5)$$

In Figure 8, the thick and thin lines correspond to upper and lower mantle depths, respectively. The horizontal bars represent the total distance between the models. As expected, the relative error is greater in the lower mantle and increases with decreasing wavelength. The peak distance occurs at degree 12 where it represents 25% of the model amplitude in the upper mantle and 50% in the lower mantle. The average distance is 17% in the upper mantle and 32% in the lower mantle.

Figures 9 and 5 & 6 (f) represent the spectral leakage in the degree 12 model, obtained by subtracting the model in Figure 3 from the model in Figure 4. The small scale structure component of the model (degrees 13-32) maps preferentially at high degrees and in the transition zone. We also note that the effect of spectral leakage on the model exceeds that of the inherent error. The maps at different depths show spurious heterogeneity that exceeds 1% in the transition zone. Figure 8 (middle) shows the dependence on wavelength of the spectral leakage, computed by taking the distance (5) between aliased and unaliased models. Here again, the peak distance occurs at degree 12, and is significantly stronger than the inherent error, representing 55% of model amplitude in the upper mantle and 70% in the lower mantle. The average distance is 28% in the upper mantle and 45% in the lower mantle.

Finally, Figures 5 & 6 (g) represent, in the absence of noise, the effects of the total error in the models, computed by subtracting model 1 from model 3. Radial smearing is noticeable at the surface and the amplitude of the error at high degrees near the 670 km discontinuity is comparable to that of the input structure (compare the peak at degree 10 in the transition zone in Figures 6 (a) and 6 (g)). Figure 8 (bottom) shows a distance between input and aliased models of 35% in the upper mantle and 50% in the lower mantle. As expected, the total error peaks at high degrees, where it represents 60% and 70% of model amplitude in the upper and the lower mantle respectively.

3.2. Comparison with tomography

While *GEMLAB 1* only represents the current status of "realistic" convection computations, it is interesting to discuss how well its "aliased" version compares to a tomographic model obtained with real seismic data and the same geometry of paths and waveforms as used in the previous synthetic experiments. As mentioned earlier, the damping parameters used for the inversion of the aliased model are optimized for our source-receiver distribution and for waveforms that have sampled a mantle model containing short wavelengths. The prejudice they impose on the inversion is indeed that which best recovers degrees 0 through 12 of the synthetic input structure and therefore also represents the best we can do for the inversion of actual data for heterogeneity up to degree 12, under the assumption that recovering a model as close as possible to the convection model *GEMLAB 1* is a desirable goal.

Since model *SAW12D* was originally obtained using a different set of damping parameters, in order to compare more effectively the inversion results using actual data to the synthetic inversions performed here, we compute a new tomographic model based on our actual dataset, using the optimized set of damping parameters determined in the calculation of the aliased *GEMLAB 1*. In this manner, we cannot completely confirm the reality of the features of the *GEMLAB 1* model that can be recovered through seismic tomography, but we can assess those features in the tomographic models that are significantly different from the predictions of the convection computation. An element of circularity is also present in calibrating smoothness constraints based on convection calculations made to match the tomography. To check whether this method could introduce features of the reference model where they do not exist, [*Mégnin et al.*, 1997] computed a set of synthetic seismograms in a convection model including a phase transition at 670 km, and derived a set of damping parameters adjusted to optimize the recovery of the input model. The inversion of this data set showed good restitution of the spectral footprint of the phase transition. A second set of synthetics was traced in another convection model which did not include the phase transition and was inverted with the same tuning parameters. The inversion of this new set of synthetics did not introduce the pattern associated with the phase transition in the control model, and hence suggests that there is no effective circularity in this methodology.

Figure 10 is a mantle model derived with damping parameters that best allow the recovery of the heterogeneity pattern of *GEMLAB 1*; it is readily compared to the aliased convection calculation of Figure 3. The models show both similarities and differences across the mantle. Notable differences are slab structures, which display more continuity in the convection simulation (*e.g.* under South America) In particular in the mid-mantle (1,600 km), the expectations from plate subduction show strong differences with the observations. Clearly, this discrepancy is not due to the tomography since continuity of slab structure is observed in the 'aliased' calculation. Large differences are also observed in D", where the convection calculation fails to reproduce the slow anomalies in the regions beneath the Central Pacific and Africa, probably due to the presence of chemical heterogeneity at the base of the mantle.
In Figure 11, we show a spectral comparison between the aliased model (left) and the image resulting from the optimized inversion of real data (center). The right of the plot shows the rms profiles for *SAW12* modified (solid line) and *GEMLAB 1* aliased (dashed line). The SHMs have been truncated at a minimum depth of 200 km because the convection model was not constructed with a realistic lithosphere.
An important similarity between the two models is in the transition zone where the spectral signature of the phase transitions in *GEMLAB 1*, although somewhat attenuated, is well reproduced in the tomographic pattern, up to degree 5. In both models, the heterogeneity at degrees 2 and 3 is largely unaffected by the phase transition and penetrates into the lower mantle down to 800 km depth. Convection simulations by [*Bunge et al.*, 1997; *Tackley et al.*, 1993] show that the introduction of a phase transition in the convection simulation increase the long wavelength amplitudes of the heterogeneity pattern in the transition zone. Based on the comparison between the two models, the convection calculation seems to overpredict the Clapeyron slope of the phase transition. Contingent upon the correct modeling of the time dependence of the subduction history, this observation suggests that the strength of the phase transition is less than $-4MPa.K^{-1}$.

Both models predict a strong pattern in the lower mantle at degree 2 and, in the D" region, at degrees 2 and 3. The tomography nonetheless shows considerably more complexity with heterogeneity at higher degrees, in particular in the lowermost mantle. In the lower mantle, the convection calculation shows a spectral peak at degree one which is not observed in the seismic images. Another important difference is the absence of the two large upwellings under Africa and the Pacific observed in all recent tomographic images of the whole mantle. This difference may be due to the presence of effects such as partial melting or other chemical anomalies in the lowermost mantle that have not been modeled in the convection simulation. It thus appears that, while current generation convection models seem to be relatively successful in modeling downwellings, future calculations must deal better with the characteristics of the upwellings as shown by tomography. This suggests that the introduction of yet unmodeled physical parameters such as rheological and/or chemical stratification (*e.g.* [*van der Hilst and Kárason*, 1999]), as well as partial melting and temperature-dependent viscosity may be necessary in order to bring better agreement between models and observations.

4. DISCUSSION AND CONCLUSION

In the present study, we used convection simulations to calibrate the recovery of heterogeneity characteristic of the mantle's convective pattern by seismic imaging techniques. We have applied a tomographic filter to results from 3-D spherical convection calculations that incorporate the history of plate motions starting in the Mid-Mesozoic, a viscosity contrast between upper and lower mantle, phase transitions at 410 km and 670 km, and 20% heating from the core. This model represents one of the most realistic convection computations currently available, in that it predicts a spectrum of heterogeneity peaked at low degrees in the lower mantle. The filtered convection model is compared to results derived from the inversion of actual waveform data, using an optimized set of damping parameters. A comparison between the two models allows to make the following inferences regarding the *GEMLAB 1* model:

1) The spectral peak at the low degrees in the pattern of heterogeneity is continuous from the transition zone to depths of about 800 km, across the 670 km discontinuity in both models.

2) In the lower mantle, the tomography shows a broader spectrum than is predicted by the convection simulation.

3) The large difference between the convection calcu-

lation and the tomography in the lowermost mantle indicates that the former must lack an important ingredient for correctly predicting the character of major upwellings. Part of the difference may be due to the constraints placed on the position of fossil slabs in the Mesozoic, but the thermal and chemical boundary conditions in the lowermost mantle do not yet seem to be well understood, and they must play an important role in the global dynamics.
4) Given that the convection calculations were made under assumption of a uniform mantle chemistry across the 670 km discontinuity, the images of transition zone structure are inconsistent with a Clapeyron slope larger than 4 $MPa.K^{-1}$. This conclusion is contingent upon the quality of the historical plate reconstruction.

The upper bound on the strength of the phase transition is also contingent on the choice of the second Grüneisen parameter g (equation 2) that we use to convert the temperature field of the convection model into shear velocity perturbations. The pressure dependence of this parameter is poorly constrained experimentally. Nevertheless, for the radial thermal expansivity profile of $PREM$, the value of $4.0K^{-1}$ chosen here for g produces a thermal field converted to velocity whose rms is in general agreement with that of of the tomography (see Figure 11, right).

Taking the difference between images of the long and short-wavelength expansions of the convection simulation, we quantify the effect of aliasing of small scale structure into long wavelength seismic images. We find that the amplitude of the spurious structure rarely exceeds 1% but is present at all depths, particularly at the shorter wavelengths. This effect is small when compared to the amplitude of the models in the upper mantle. It is nevertheless closer to the amplitude of the heterogeneity in most of the lower mantle (D" excepted) as seen in tomographic models.

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Figure 1. Convection model GEMLAB 1 expanded up to spherical harmonic degree 32.

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Figure 2. Convection model GEMLAB 1 expanded up to spherical harmonic degree 12.

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Figure 3. GEMLAB 1 aliased: Inversion of synthetics traced through the model in figure 1 for lateral heterogeneity up to degree 12.

Figure 3. GEMLAB 1 aliased: Inversion of synthetics traced through the model in figure 1 for lateral heterogeneity up to degree 12.

Figure 4. GEMLAB 1 unaliased: Inversion of synthetics traced through the model in figure 2 for lateral heterogeneity up to degree 12.

Figure 4. GEMLAB 1 unaliased: Inversion of synthetics traced through the model in figure 2 for lateral heterogeneity up to degree 12.

Figure 5. Spectral analysis of aliasing: a) Input model GEMLAB 1 (degrees 0-12). b) Inversion of synthetic data through GEMLAB 1 expanded up to degree 32. c) Inversion of synthetic data through GEMLAB 1 truncated at degree 12. d) rms profiles for the models in a) solid line -input model-, b) dotted line -aliased model-, c) dashed line -unaliased model-. e) Inherent error: difference between c) and a). f) Aliasing of degrees 13-32: difference between c) and b). g) Total distortion: difference between b) and a). h) rms profiles for e) dashed line -inherent error-, f) dotted line -aliasing-, and g) solid line -total distortion-.

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Figure 6. Same as figure 5 but the maps are normalized by the rms amplitude at each depth.

Figure 6. Same as figure 5 but the maps are normalized by the rms amplitude at each depth.

Figure 7. Spurious structure in the models caused by inherent error: absolute value of the difference between input model truncated at degree 12 and the unaliased model (Figures 5 & 6 e)).

Figure 7. Spurious structure in the models caused by inherent error: absolute value of the difference between input model truncated at degree 12 and the unaliased model (Figures 5 & 6 e)).

Figure 8. Top: Spherical harmonic degree by degree euclidian distance between unaliased and input model, relative to the unaliased model. Thick lines: upper mantle, thin lines: lower mantle. The horizontal lines represent the average distance. Middle: same as top, but for the difference between aliased and unaliased models. Bottom: Same, but for the difference between aliased and input model.

Figure 8. Top: Spherical harmonic degree by degree euclidian distance between unaliased and input model, relative to the unaliased model. Thick lines: upper mantle, thin lines: lower mantle. The horizontal lines represent the average distance. Middle: same as top, but for the difference between aliased and unaliased models. Bottom: Same, but for the difference between aliased and input model.

Figure 9. Spurious structure in the models caused by aliasing: absolute value of the difference between aliased and unaliased models (Figures 5 & 6 f))

Figure 9. Spurious structure in the models caused by aliasing: absolute value of the difference between aliased and unaliased models (Figures 5 & 6 f))

Figure 10. Tomographic model SAW12 modified.

Figure 10. Tomographic model SAW12 modified.

Figure 11. Spectral comparison between left: GEMLAB 1 aliased, and right: tomographic model SAW12 modified.

Figure 11. Spectral comparison between left: GEMLAB 1 aliased, and right: tomographic model SAW12 modified.

Table
1.
Body
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SEISMIC IMAGING OF CONVECTION MODELS

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