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Are harmonic tremors self-excited thermoacoustic oscillations?

F.H. Busse [a,*], Peter A. Monkewitz [b], M. Hellweg [c]

[a] Institute of Physics, University of Bayreuth, D-95440 Bayreuth, Germany
[b] Laboratoire de Mécanique des Fluides, Ecole Polytechnique Fédérale de Lausanne, CH-1015 Lausanne, Switzerland
[c] Berkeley Seismological Laboratory, University of California, Berkeley, CA 94720-4760, USA

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Abstract

The thermoacoustic instability is proposed as a possible cause of volcanic harmonic tremors. Properties of this instability are discussed in relationship to conditions in active volcanoes, and directions of possible future research are outlined.

Keywords: volcanic tremors; thermoacoustic instability; Sondhauss tube; thermoacoustic engine

1. Introduction

Harmonic tremors are nearly periodic oscillations with frequencies of the order of 1 Hz that occur in some volcanoes and can persist for periods as long as several hours. These oscillations are usually recorded with seismometers. They often occur in connection with eruptions, but they are also found when the volcano does not exhibit other signs of unusual activity. For a recent review of the observational evidence and theoretical models of the origin of tremors, we refer to Konstantinou and Schlindwein [1].

Volcanic tremors of various types have been observed and different theoretical models may be needed to understand their origins. In this short letter, we wish to focus the attention on *harmonic* tremors. While some such tremors seem to occur as the result of more and more rapidly occurring pulses (i.e. [2]), others follow as the coda of an explosion (i.e. [3]). Finally, some harmonic tremors simply emerge from and disappear into the seismic background with no apparent triggering cause (i.e. [4]). Tremors of this type should be most readily accessible to theoretical modelling because of their regular nature. The simplicity of the phenomenon in an environment as complex as a volcano is a major challenge for theoreticians. Several models proposed in the past have associated harmonic tremors with the acoustic eigen modes of large gas bubbles or gas filled cavities, but the source of the excitation of those oscillations has remained obscure.

In order to keep the excitation in phase with the oscillations, even when the frequency drifts as is often observed (see, for instance, [5,4]), both must be intimately connected. From this point of view, the thermoacoustic instability is a particularly attractive mechanism of excitation.

Spontaneous acoustic oscillations induced by temperature gradients have been discussed in the scientific literature for more than two centuries and Lord Rayleigh [6] has devoted a section in Volume II of his

* Corresponding author.
E-mail address: busse@uni-bayreuth.de (F.H. Busse).

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famous treatise "The Theory of Sound" to this phenomenon. He had a clear physical understanding of the mechanism when he states "...the effect will depend on the phase of the vibration at which the transfer of heat takes place. If heat is given to the air at the moment of greatest condensation, or be taken from it at the moment of greatest rarefaction, the vibration is encouraged." A detailed theoretical understanding has been achieved only rather recently. The boundary layer theory developed by Rott [7] and extended by him and his coworkers (see the review by Rott [8] for references) was the first to provide quantitative predictions which compared reasonably well with experimental measurements.

Thermoacoustic oscillations play a role in several fields of engineering. They are usually regarded as a nuisance when they appear in the operation of cryogenic equipment and engineers often must change their designs to avoid them. On the other hand, thermoacoustic oscillations provide a heat transport mechanism for refrigerators without moving parts. For a review of thermal engines based on thermoacoustics, we refer to the article by Swift [9]. It is remarkable that Stirling engines based on thermoacoustic oscillations can achieve much higher efficiencies in converting thermal into mechanical energy than internal combustion engines. The not yet optimally designed laboratory prototype of a thermoacoustic Stirling engine delivered already 700 W of acoustic energy corresponding to 30% of the heat input which in turn corresponds to 40% of the Carnot efficiency [10].

In the following we shall discuss a number of issues related to the proposal that the process of conversion of heat into acoustic energy may operate in volcanoes and manifest itself in the form of harmonic tremors. Since little is known about the interiors of volcanoes, it is not possible to construct detailed models for thermoacoustic oscillators and the arguments given below are of a qualitative rather than a quantitative nature. Nevertheless, we hope to convince the reader that the thermoacoustic instability must be regarded as a serious contender for the origin of harmonic tremors.

2. General considerations

The frequent occurrence of thermoacoustic oscillations in cryogenic equipment is due to the fact that the ratio of temperatures at different ends of a helium filled cavity can easily exceed a factor of 2, while such ratios (based on the Kelvin scale) are not so easily encountered under normal laboratory conditions. The temperature ratio α between the hot and the cool parts of a gas filled cavity is usually introduced as the primary parameter in theories of the thermoacoustic instability [8]. But the typical critical value of $\alpha_c \approx 2$ can be lowered considerably if additional effects are taken into account, such as the presence of condensation and evaporation [11], or if the frequency is optimized by coupling the oscillator to a suitable mass of liquid [12]. In the neighborhood of liquid magma inside a volcano, temperature changes of the order of a few hundred degrees over a distances of several meters must be expected, especially if layers saturated with water exist nearby. The cooling of magma penetrating into parts of a cavity of the size of several meters should therefore be able to sustain thermoacoustic oscillations over the time span of hours.

To support this statement one can estimate the pressure oscillation amplitude of a gas filled spherical cavity that is required to generate the observed tremors. This has been done by Hellweg [13] for Lascar Volcano. She finds that a radius of 20 m is required when a maximum pressure difference between cavity and its environment of 7 bar is applied. Since thermoacoustic oscillations often saturate at oscillating pressure amplitudes comparable to the mean pressure [9], it seems possible that harmonic tremors could be excited in cavities at a depth of the order of 100 m or more. This may be an overestimate, however, as we shall point out in Section 5 below.

3. Dynamic considerations

The frequency of acoustic oscillations in a cavity filled with a compressible fluid corresponds approximately to the basic eigen mode of the cavity. The association between volcanic tremors and the eigen oscillations of large gas bubbles or gas filled cavities is a basic element of several tremor theories [1].

The eigen frequency of a closed tube of length ℓ filled with an ideal gas is $f_0 = \sqrt{\gamma p/\rho_g}/2\ell$ where $\gamma = c_p/c_v$ is the ratio of the specific heats, p is the mean pressure and ρ_g is the mean density of the gas. The basic frequency of a cavity of largest dimension ℓ usually does not differ much from this value. This implies that under normal conditions with a pressure of $p = 10^5$ Pa and a temperature of $T = 300$ K a length $\ell = 172$ m is needed to obtain a frequency of 1 Hz. Such a cavity length appears to be a bit too large to be expected inside volcanoes, although lengths of that order of magnitude have been proposed in some theories. But there are many possibilities for obtaining acoustic oscillations with frequencies of 1 Hz in smaller cavities. A Helmholtz resonator, for instance, consisting of an open tube of length ℓ with a cross-sectional area A which is attached to a cavity with the volume $V \gg A\ell$, oscillates with the frequency $f_1 = c\sqrt{A/V\ell}/2\pi$ where c denotes the speed of sound, $c = \sqrt{\gamma p/\rho_g}$. In this situation the inertia is provided only by the gas in the tube while the cavity acts as a rather soft spring. The frequency is thus reduced by the square root of the ratio of tube volume to cavity volume. A further decrease of the frequency is obtained when the outer part of length s of the horizontal tube is filled with a liquid of much higher density ρ_1 than that of the gas. Here the expression $f_2 = \sqrt{\gamma pA/\rho_1 sV}/2\pi$ is obtained for the frequency [12] which represents a reduction by a factor of about $\sqrt{\rho_g/\rho_1}$. Gravity g enters the picture when the liquid filled part of the tube is not horizontal, but forms a vertical U-tube. Here the frequency becomes $f_3 = \sqrt{f_2^2 + (1+S)g/4\pi^2 s}$ [14], where s is the length of the liquid column and S is the ratio of the free surface areas at its inner and outer ends.

These considerations can give only a rough idea of the types of oscillations that can set in spontaneously through the thermoacoustic mechanism. The actual values of the frequencies are modified by dissipative effects, especially those associated with the excitation mechanism, for which we refer to the literature [8,15]. It is worth mentioning that purely liquid thermoacoustic oscillators have also been realized. Liquids with high thermal conductivities and low viscosities such as liquid metals are especially suitable and the experiment reported by Swift [9] has been carried out with liquid sodium. However, magma filled cavities without a second fluid are unlikely candidates for the thermoacoustic instability because of the relative high viscosity, the high Prandtl number, and because of the relatively large cavity dimension required.

There are numerous spectral analyses of harmonic tremor recordings available in the literature. The spectra exhibit strong higher harmonics and overtones up to the 25th have been seen [4]. Unfortunately it is difficult to exploit the spectral information in theoretical models since the seismic spectra not only depend on the nonlinear property of the acoustic oscillation, but also on the properties of the cavity walls and other parameters. An interesting characteristic of some recordings of harmonic tremors is that odd harmonics have diminished amplitudes in comparison with the even ones [5,16]. Sometimes the first harmonic appears to be suppressed entirely. Such features can also be found in studies of the dependence of the damping of thermoacoustic oscillation on the geometrical configuration of the cavity. Karpov and Prosperetti [15], for example, find that odd modes are much more strongly damped than even ones when the tube like cavity has a constriction in the middle.

4. Excitation mechanism

The amplification of the acoustic oscillation is achieved when the fluid receives more heat from the cavity wall in the compression phase than in the expansion phase. An effective thermal contact with a heat reservoir at a suitable place in the cavity is most important. In thermoacoustic engines [9] stacks of heated plates are placed parallel to the oscillating velocity in the hot part of the cavity. The distance between the plates is a low multiple of the thermal boundary layer thickness $\delta_T = \sqrt{\kappa/2\pi f}$ where κ is the thermal diffusivity of the gas. Since experimental thermoacoustic engines usually operate at frequencies of a few hundred Hertz, a gap of the order of a few mm between the plates is optimal. If the gap becomes too small viscous dissipation will increase and acoustic oscillation are choked off. Theoretical expressions for the onset of thermoacoustic instability in the absence of a stack of plates indicate a decreasing critical temperature ratio α_c with decreasing frequency because of the enhancement of the heat transfer between oscillating fluid and solid wall. An increase of the effective boundary layer thickness must also be expected along a rough wall or when the flow close to the wall becomes turbulent. But these topics do not seem to have been studied yet in the case of thermoacoustic oscillations.

As an example among the various configurations of thermoacoustic oscillators in volcanoes, we consider a Sondhauss-tube like system as sketched in Fig. 1. The volume V of hot gas acts as the spring in this case, while the water in the channel provides the inertia. A wide range of eigen frequencies are possible depending on the system parameters. The major part of the heat transfer from the magma occurs near the narrow neck where also the strongest temperature gradient is realized. The conditions for an efficient excitation of the

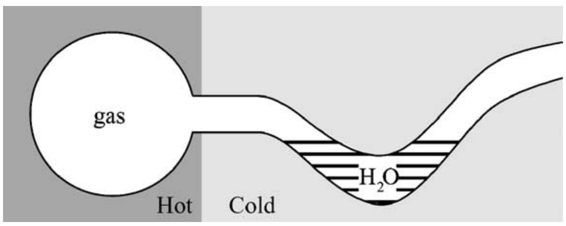

Fig. 1. Qualitative sketch of a thermoacoustic source of harmonic tremors.

thermoacoustic instability are thus given according to the theory of Rott [8]. The property that oscillation frequencies of the order of 1 Hz seem to be preferred in volcanoes may be due to the fact that while low frequencies enhance the onset of instability, they may lead to fully turbulent flow when they drop significantly below 1 Hz.

In volcanoes the excitation of harmonic tremors occurs in several ways. Often they evolve as slowly growing oscillations without a particular onset [4] as one might expect of an instability that is induced when a critical value of a control parameter is exceeded owing to gradually changing conditions. The gradual shift with time of frequencies, called "gliding" in the case of harmonic tremors, is a typical feature that can also be expected in the case of thermoacoustic oscillations when, for instance, the position of maximum heat input changes slowly or when the mass of the water participating in the oscillation varies gradually. Abrupt changes could lead to tornillos which are characterized by a sudden onset with a distinct frequency, but exhibit a nearly exponential decay as their source of energy is rapidly depleted.

5. Energetic considerations

In this section we derive an expression for the energy radiated away from a radially oscillating spherical cavity and use it for an estimate of the power provided by a thermoacoustic tremor source in a volcano. Following the analysis described in chapter 4 of the book by Achenbach [17], we find that the radial displacement u in the elastic space surrounding a spherical cavity of radius a oscillating with the angular frequency ω is given as function of time t and distance r from the center of the sphere by

$$u(r,t) = \mathrm{Re}\left\{\frac{A}{r}\left(\frac{i\omega}{C_L} - \frac{1}{r}\right)\exp\left[i\omega\left(\frac{r-a}{c_L} - t\right)\right]\right\}, \tag{1}$$

where "Re" denotes the real part, c_L the longitudinal wave speed and A measures the amplitude. The corresponding radial stress τ_r is given by

$$\tau_r(r,t) = \mathrm{Re}\left\{-\rho\frac{A}{r}\left(\omega^2 + 2i\omega\eta\frac{c_L}{r} - 2\eta\frac{c_L^2}{r^2}\right) \times \exp\left[i\omega\left(\frac{r-a}{c_L} - t\right)\right]\right\}. \tag{2}$$

In this equation, the definition $\eta=(1-2v)/(1-v)$ has been used with v denoting the Poisson ratio. The energy P radiated from the oscillating cavity per unit time is given by the integral of the time average of the product $-\tau_r \partial u/\partial t$ over any spherical surface of radius $r>a$,

$$P = -4\pi r^2 \overline{\tau_r \partial u(r,t)/\partial t} = 2\pi\rho\frac{A^2\omega^4}{c_L}. \tag{3}$$

When measurements of tremors of Lascar Volcano obtained in 1994 [13] are used, P can be estimated. Using $\omega=2\pi\cdot 0.63$ Hz, $c_L=1000$ m/s, $\rho=2700$ kg/m^3, $v=0.3$ and the measured rms displacement of about $4\cdot 10^{-8}$ m at the distance $r=4000$ m from the tremor source, we obtain a value for A of the order of 0.06 m^3. This corresponds to an amplitude of 1.8 bar of the oscillating component of the pressure in a cavity with radius $a=10$ m. For smaller radii higher pressure amplitudes are obtained since the latter vary essentially with a^{-3}. For the corresponding radiated power P a value of only about 14 W is obtained. This value of P for a cavity of 20 m diameter appears to be rather low in view of the power obtained in experiments on a laboratory scale. It should be realized, of course, that the inclusion of the power radiated in the form of higher harmonics may well increase the estimate by an order of magnitude. We have also not taken into account dissipative energy losses which are difficult to estimate. On the other hand, the tremor energy may be radiated preferentially in the horizontal directions. Our main conclusion is thus that energy considerations do not exclude the possibility that harmonic tremors are generated by self-excited thermoacoustic oscillations in at least some volcanoes.

6. Concluding remarks

We close this brief report without developing a more detailed model for thermoacoustically driven harmonic tremors. The present state of knowledge does not seem to justify the derivation of more specific expressions for thermoacoustic oscillations than those already available in the literature. Hence, we just summarize the arguments that have led us to propose thermoacoustic oscillations as a viable mechanism for the generation of harmonic tremors in the interior of volcanoes.

- Harmonic tremors exhibit the characteristic properties of spontaneously excited instabilities, such as the thermoacoustic instability.
- Thermoacoustic oscillations are known to be particularly efficient converters of thermal energy into acoustic energy. They satisfy the energetic demands for tremor generation.
- The coupling of excitation mechanism and oscillator that characterizes thermoacoustic oscillators pro-

vides a most natural explanation for the high Q value and the long-term coherence of harmonic tremors.
- Several other properties of harmonic tremors can also be found in thermoacoustic systems such as gliding frequencies and strong overtones.

Obviously, further research in several directions could clarify some of the issues mentioned above. Experiments on acoustic oscillations in more complex cavities than the commonly used straight tube or the simple Helmholtz resonator (Sondhauss tube) are desirable in particular with regard to the frequency spectra in the nonlinear regime of oscillations. The influence of rough walls and the effects of turbulent boundary layers would also be interesting topics of experimental research.

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