Calculation of normal modes for Earth model with slight rigidity in the liquid outer core

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Abstract. We calculate normal modes for the Earth model, which has a slight rigidity layer at the base of the liquid outer core. We show that such a layer with thickness about 100 km and the shear wave velocity of 0.005 km/sec can produce a normal mode, which has a close eigenfrequency to that of liquid core model, without affecting fundamental modes and most of the higher modes. Our results indicate that the thin finite rigidity layer in the outer core might explain the anomalous splitting of the Earth's normal modes.

1 Introduction

The nature of an anomalous splitting of the Earth's normal modes, which is characterized as a spectral splitting width of several normal modes is almost twice as large as that is predicted by the rotation and hydrostatic ellipticity of the Earth (Masters and Gilbert, 1981; Ritzwoller *et al*, 1986; Giardini *et al*, 1987), has been controversial for more than a decade (Dahlen and Tromp, 1998). It is now considered that the anomalous splitting can be explained by the inner-core anisotropy (Woodhouse *et al*, 1986; Tromp, 1993; Dahlen and Tromp, 1998). However, there still remain core sensitive modes which are not fully explained by the inner-core anisotropy. Based on this observation, it is argued that there exists a heterogeneous structure in the Earth's liquid outer core (Widmer *et al*, 1992; Romanowicz and Breger, 1999; Romanowicz *et al*, 2000). Although it is difficult to introduce heterogeneity of compressional wave velocity and density in the liquid outer core, finite rigidity in the outer core might become an alternative model to inner-core anisotropy. Sato (1964) has shown that if we allow slight rigidity in the liquid core, the modes which have nearly equal eigenfrequencies to that of a liquid core model would appear. Since this is similar to the spectrum splitting of normal modes, he called this as "soft core splitting". Here, we consider the Earth model which has an extremely slow shear velocity layer at the bottom of the liquid outer core. We show that such a layer with thickness about 100 km and the shear wave velocity of 0.005 km/sec can actually produce soft core splitting modes for core sensitive modes, such as ${}_3S_2$, without affecting fundamental modes and most of the higher modes. This shows that small finite rigidity in the outer core can be considered as an alternative model to explain anomalous splitting of normal modes.

2 Soft core splitting

It is generally considered that the anomalous splitting of the normal modes is explained by the inner-core anisotropy (Woodhouse *et al*, 1986; Tromp, 1993; Dahlen and Tromp, 1998). This is mainly because the inner-core anisotropy obtained from normal modes seems to be consistent to those obtained from travel time observations (Poupinet *et al*, 1983; Creager, 1992). However this consistency is obtained at the expense of fit of anomalously splitted normal modes, such as ${}_3S_2$. The splitting width of ${}_3S_2$ predicted by the inner-core anisotropy is much larger than that of the observation. Since this mode is sensitive to the structure of the outer core, it is argued that there may be some heterogeneous structure of the compressional velocity or the density in the liquid outer core (Widmer *et al*, 1992; Romanowicz and Breger, 1999; Romanowicz *et al*, 2000), although the general idea that the liquid outer core should be homogeneous has ruled out this possibility.

Heterogeneous structure in the outer core is required if we should explain the anomalous splitting by the splitting of isolated normal modes. There may be another possibility that spherically symmetric structure in the outer core may produce splitting-like behavior of normal modes. Sato (1964) has calculated torsional oscillation of an earth model consists of a homogeneous mantle and a slightly rigid homogeneous core. He showed that if there is a finite rigidity in liquid core with several percent of the mantle, there appear eccentric modes which have nearly equal frequencies to those of a liquid core model. Since these modes appear in the spectrum with nearly equal amplitudes and frequencies, this may be considered as a kind of spectrum splitting. Sato has called these phenomena as “soft core splitting”.

3 Earth model with slight rigidity in the outer core

Finite rigidity with several percent of the mantle should be impossible to be introduced in the Earth's liquid core, since the agreement of the observed eigenfrequencies of fundamental and higher modes with those predicted by the spherically symmetric Earth model with liquid outer core is excellent (Gilbet and Dziewonski, 1975; Dziewonski and Anderson, 1981). However, it may be possible to have a thin finite rigidity layer in the liquid outer core without affecting most of the normal modes except those sensitive to outer core structure. It is shown that the anomalous splitting is not observed for torsional modes (Tromp and Zanzerkia, 1995), which implies that finite rigidity layer should not exist below the core-mantle boundary. Here we calculate spheroidal modes for Earth model with slight rigidity in just above the inner core boundary and examine if there exists “soft core splitting”. We use isotropic PREM (Dziewonski and Anderson, 1981) as spherically symmetric Earth model and introduce slight shear wave velocity at the lowermost layer of the outer core.

4 Result for ${}_0S_2$

We use program package DISPER80 (Saito, 1988), which uses Runge-Ktta-Gill method, to obtain eigenfrequency and eigenfunction of “soft core” modes. We first show the upper limit of finite rigidity, or slight shear wave velocity, in the outer core. We calculate the eigenfrequency and the eigenfunction of ${}_0S_2$ for “soft core” Earth model with the layer thickness 126 km and constant shear wave velocity of 0.005 km/sec in the bottom of the liquid outer core. The radial eigenfunctions of this ${}_0S_2$ equivalent mode are shown in Figure 1. The period of this mode is 3233.26 sec, while that predicted by isotropic PREM with a liquid core is 3233.45 sec and the observation is 3233.25 sec. The radial eigenfunction oscillates severely within the finite rigidity layer, but the period of this mode does not differ significantly from both observation and that predicted by the liquid core model. There is no soft core splitting mode for ${}_0S_2$. Thus, this order of finite rigidity in the liquid outer core does not affect ${}_0S_2$.

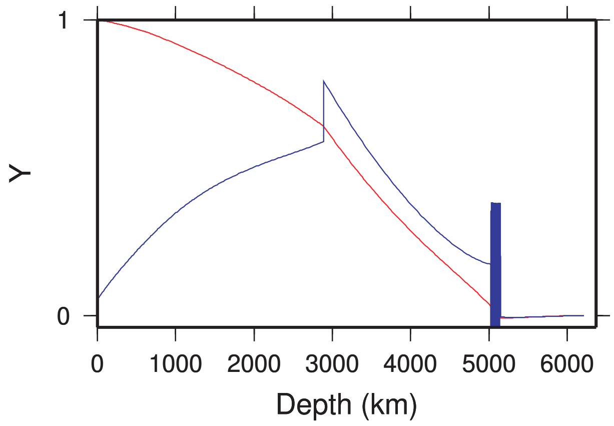

Fig. 1. Radial eigenfunctions of ${}_0S_2$ for soft core Earth model. Red: vertical displacement $y_1(r)$ of Saito (1988), Blue: horizontal displacement $y_3(r)$ of Saito (1988). Discontinuites in the eigenfunctions indicate the core- mantle boundary (at depth 2891 km) and the inner core- outer core boundary (at depth 5149.5 km). The period of this mode is 3233.26 sec.

Fundamental spheroidal modes with the angular order greater than 8 (period 707 sec) do not have sensitivity to the inner core boundary structure. We have calculated fundamental modes up to the angular order 8 and found that the eigenfrequencies are not affected and there are no soft core splitting modes. We also have calculated the first higher spheroidal modes up to the angular order 6 (period 657 sec) for the same soft core Earth model. There are no soft core splitting modes for these higher modes, which shows that these first higher modes are not affected by slight shear wave velocity layer at the bottom of the outer core. We may conclude that this order of finite rigidity (shear wave velocity 0.005 km/sec, thickness 126 km) may be regarded as the upper limit that can be introduced at the bottom of the liquid outer core. If we increase the shear wave velocity to 0.01 km/sec, for example, then the eigenfrequency of ${}_0S_2$ deviates significantly from that of the liquid core model. The thickness of 126 km is arbitrarily chosen from the radial mesh of the original model parameter. So the upper limit of thickness may vary, but we have confirmed that the layer thicker than 160 km will affect the eigenfrequency of ${}_0S_2$.

5 Result for ${}_3S_2$

We have calculated, then, the normal modes of anomalously splitted spheroidal modes for the same soft core Earth model. We have found that there exist two soft core splitting modes for ${}_3S_2$ as Sato predicted. The radial eigenfunctions of these modes are

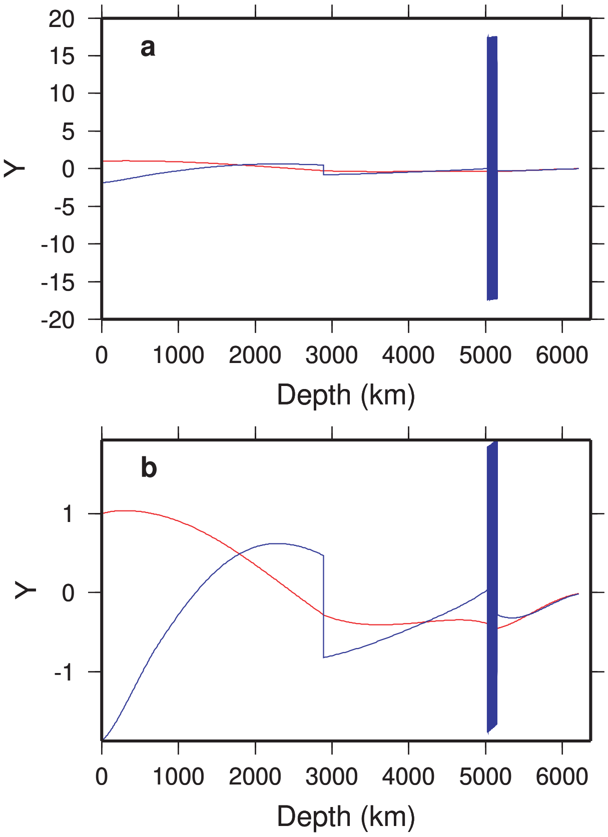

Fig. 2. Same as figure 1 but for ${}_3S_2$ for soft core Earth model. The period of the mode is (a) 905.16 sec and (b) 903.87 sec.

shown in Figure 2. One mode has a similar shape of eigenfunction to that of the liquid core model with the period of 903.87 sec. The other has basically the same eigenfunction but large amplitude within the finite rigidity layer in the outer core with the period of 905.16 sec. There are only these two soft core splitting modes for ${}_3S_2$ equivalent mode. The anomalous splitting width of ${}_3S_2$ is reported to be between about 904 sec and 912 sec. The liquid core model predicts the period should be 904 sec and the splitting width due to the Earth's rotation and elliptical figure should be between about 902 and 907 sec. Since the eigenfunctions of soft core splitting modes have the same characteristics as those of the liquid core model, we may assume that they have the same rotational and elliptical splitting parameters. Then the total splitting width of these soft core splitting modes may be between 902 and 910 sec, which gives close agreement to the observation. The amplitude of eigenfunction within the finite rigidity layer of the mode with the period of 905.16 sec is about one twentieth to the amplitude near the Earth's surface. It is not necessarily large but is sufficient to be excited by the earthquakes that occurred near the Earth's surface.

6 Conclusion

We have calculated normal modes for the Earth model which has slight rigidity in the bottom of the liquid outer core. Our preliminary calculation suggests that the slight rigidity layer with the thickness 126 km and the shear wave velocity of 0.005 km/sec is compatible with the fundamental modes and most of the higher modes. We found that core-sensitive mode, such as ${}_3S_2$, has soft core splitting modes, which may explain anomalous spliting. This result indicates that the slight rigidity in the liquid outer core can be considered as an alternative model of anomalous splitting to the inner core anisotropy.

7 Reference

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