Linear Response of the outer core flow to thermal heterogeneities on the core-mantle boundary

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Abstract.

We investigate the linear response of the outer core fluid to the temperature heterogeneity on the CMB (core-mantle boundary). We examine non-magnetic response both analytically and numerically. Analytic solutions are obtained for the two limiting cases of large and small Ekman numbers. Numerical calculations are used to connect the two limits. We find that the longitudinal phase shift of the response is slightly westward relative to the non-rotational case for weak rotation, and $\pi/2$ eastward for strong rotation, irrespective of the pattern of the thermal heterogeneity. These results can be understood in terms of the vorticity balance. The Ekman number for the transition between the two regimes is about 10^{-2} for the heterogeneities of the harmonic degree two. These results will serve as a basis for interpreting the pattern of the geomagnetic field in terms of thermal core-mantle interations.

1 Introduction

It is important to obtain information in the Earth's core for the understanding of the dynamics in the Earth's deep interior. However, our knowledge about the flow is still limited due to the lack of direct observations. The most important observation about the flow is the geomagnetic field, because the field is generated by a fluid motion of the liquid iron in the outer core. The information is, however, limited because what we can observe is the field pattern only on the core surface. One of the idea for overcoming this limitation is to study core-mantle coupling, because we have seismological observations about the mantle convection in addition to the geomagnetic field observations. Comparison of the observations of the core and mantle will enable us to reveal some physical properties which cannot be obtained solely from geomagnetic observations. In that respect, thermal influence of the mantle on the core flow can be a clue to the understanding of the properties of the outer core.

The geomagnetic field has components which are stationary relative to the mantle for a long time (e.g. Gubbins and Kelly, 1993). This indicates core-mantle interactions, because the field pattern would drift eastward or westward if the mantle did not influence the core flow. Therefore, stationary features should be considered as a sign of core-mantle interactions. Bloxham and Gubbins (1997) proposed thermal core-mantle interactions to be one of the most important mechanisms responsible for the stationary features.

Various studies have been made to clarify the physical mechanism of thermal interactions (e.g. Zhang and Gubbins, 1992; Yoshida and Hamano, 1993; Glatzmaier et al., 1999; Sumita and Olson, 1999; Gibbons and Gubbins, 2000; see references therein). They set some heterogeneities of temperature or heat flux on the CMB, and studied the response of the flow in the outer core. However, most of them are numerical studies, and detailed physical processes are not clarified yet. We focus on a linear problem here, and examine the underlying physical process carefully. Among previous studies, those which are closely related to our work are Zhang and Gubbins (1992), Yoshida and Hamano (1993), and Gibbons and Gubbins (2000). Zhang and Gubbins (1992) and Gibbons and Gubbins (2000) performed weakly nonlinear calculations, but they did not carry out detailed investigation on the linear problem. Yoshida and Hamano (1993) studied some cases of the linear response, but their investigation is limited to sectorial temperature heterogeneities and the quasi-geostrophic regime. We perform more detailed analysis of the linear theory in this study.

2 Linear Response Problem

We investigate the linear response of the outer core flow to the thermal heterogeneities on the CMB. We study the simplest case, which allows analytical calculation in some limiting cases. Steady response is examined without the magnetic field or the basic flow. The stratification is assumed to be neutral. The inner core is neglected. In that situation, the problem is reduced to solving the three equations

$$\begin{aligned} \nabla \cdot \boldsymbol{u} &= 0 &(1)\\ 2\boldsymbol{\Omega} \times \boldsymbol{u} + \nu\nabla^2\boldsymbol{u} + \nabla\pi &= -\alpha T\boldsymbol{g} &(2)\\ \nabla^2 T &= 0, &(3)\end{aligned}$$

where $\boldsymbol{u}$ is the velocity, π is the pressure divided by the density, T is the temperature, $\boldsymbol{\Omega}$ is the angular velocity vector of the Earth's rotation, ν is the kinematic viscosity, and $\boldsymbol{g}$ is the gravity acceleration vector. Eq (**??**) is the equation of continuity, Eq (**??**) is the equation of motion, and Eq (**??**) is the thermal diffusion equation, which is reduced to the Laplace equation in this situation. These equations are solved with the no-slip boundary condition. The tempearature at the CMB is given in terms of spherical harmonics

$$T = T_{l0}^m Y_l^m(\theta, \phi) \quad \text{at } r = a, \tag{4}$$

where a is the outer core radius, and (r, θ, ϕ) are spherical coordinates. With an appropriate non-dimensionalisation, it can easily be shown that the only non-dimensional parameter in this problem is the Ekman number

$$E = \frac{\nu}{\Omega a^2}, \tag{5}$$

which is the ratio of the internal friction to the Coriolis force.

3 Numerical Method

For numerical investigation of the problem, we expand variables by generalized spherical harmonics (Burridge, 1969; Phinney and Burridge, 1973; Mochizuki, 1988). The temperature, the pressure, and the velocity fields are transformed into the canonical coordinates, and expanded as

$$\begin{aligned} T &= \sum_{lm} T_l^m(r) Y_l^{0;m}(\theta,\phi) &(6)\\ \pi &= \sum_{lm} \Pi_l^m(r) Y_l^{0;m}(\theta,\phi) &(7)\\ u^\alpha &= \sum_{lm} U_l^{\alpha;m}(r) Y_l^{\alpha;m}(\theta,\phi) &(8)\end{aligned}$$

where the superscript α takes the values $-, 0, +$, l is the harmonic degree, m is the harmonic order, $Y_l^{\alpha;m}(\theta,\phi)$ is the generalized spherical harmonics, and (r, θ, ϕ) is the spherical coordinates. Eqs (1)-(3) become

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2 U_l^0\right) - \frac{1}{r}\sqrt{\frac{l(l+1)}{2}}\left(U_l^- + U_l^+\right) = 0 \tag{9}$$

$$\begin{aligned} &-\frac{1}{r}\sqrt{\frac{l(l+1)}{2}}\Pi_l^m \\ &+\nu\left\{\left(\frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr} - \frac{l(l+1)}{r^2}\right)U_l^{-;m} + \frac{\sqrt{2l(l+1)}}{r^2}U_l^{0;m}\right\} \\ &-i2\Omega(2l+1)(-1)^{m-1} \\ &\times\sum_{l'=l-1}^{l+1}\left[\left\{\begin{pmatrix} l & +1 & l' \\ +1 & 0 & -1\end{pmatrix}U_{l'}^{-;m} - \begin{pmatrix} l & +1 & l' \\ +1 & -1 & 0\end{pmatrix}U_{l'}^{0;m}\right\}\begin{pmatrix} l & +1 & l' \\ -m & 0 & m\end{pmatrix}\right] \\ &= 0 \end{aligned} \tag{10}$$

$$\begin{aligned} &-\frac{d}{dr}\Pi_l^m + \alpha T_l^m g \\ &+\nu\left\{\left(\frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr} - \frac{l^2+l+2}{r^2}\right)U_l^{0;m} + \frac{\sqrt{2l(l+1)}}{r^2}\left(U_l^{-;m} + U_l^{+;m}\right)\right\} \\ &-i2\Omega(2l+1)(-1)^{m} \\ &\times\sum_{l'=l-1}^{l+1}\left[\left\{\begin{pmatrix} l & +1 & 'l \\ 0 & +1 & -1\end{pmatrix}U_{l'}^{-;m} - \begin{pmatrix} l & +1 & 'l \\ 0 & -1 & +1\end{pmatrix}U_{l'}^{+;m}\right\}\begin{pmatrix} l & +1 & l' \\ -m & 0 & m\end{pmatrix}\right] \\ &= 0 \end{aligned} \tag{11}$$

$$\begin{aligned} &-\frac{1}{r}\sqrt{\frac{l(l+1)}{2}}\Pi_l^m \\ &+\nu\left\{\left(\frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr} - \frac{l(l+1)}{r^2}\right)U_l^{+;m} + \frac{\sqrt{2l(l+1)}}{r^2}U_l^{0;m}\right\} \\ &-i2\Omega(2l+1)(-1)^{m+1} \\ &\times\sum_{l'=l-1}^{l+1}\left[\left\{\begin{pmatrix} l & +1 & l' \\ -1 & +1 & 0\end{pmatrix}U_{l'}^{0;m} - \begin{pmatrix} l & +1 & l' \\ -1 & 0 & +1\end{pmatrix}U_{l'}^{+;m}\right\}\begin{pmatrix} l & +1 & l' \\ -m & 0 & m\end{pmatrix}\right] \\ &= 0 \end{aligned} \tag{12}$$

$$\left[\frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr} - \frac{l(l+1)}{r^2}\right]T_l^m(r) = 0, \tag{13}$$

where $\begin{pmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \end{pmatrix}$ is the Wigner 3-j symbol. These equations are then discretized in the radial direction. We use the first-order finite difference for the discretization.

We solve the resulting set of linear algebraic equations. We take 1000 radial mesh points, and the trancation harmonic degree is taken to be 100. Good numerical convergence is obtained down to the Ekman number of 10^{-5}.

4 Results

Fig. 1 shows the radial velocity distribution for the $(l, m) = (2, 2)$ CMB temperature pattern. The solution for the linear theory can be categorized into two regimes depending on the Ekman number E. The boundary between the fast rotation regime and the slow rotation regime is about $E = 10^{-2}$. In the slow rotation regime, the pattern shifts slightly westward as the Ekman number decreases. In the fast rotation limit, the pattern shifts eastward by $\pi/2$ in terms of phase, relative to non-rotational case. The pattern shifts westward as the Ekman number increases.

We obtain an analytical solution for each limiting case (Yoshida, in preparation), and the numerical results can be understood in terms of perturbation from the two limit. The solutions in the slow rotation regime can be understood by a usual perturbation theory. The solutions in the fast rotation regime can be understood by the superposition of the Ekman layer, the equatorial modified Ekman layer, and the Stewartson layer on the thermal wind solution. The result is an extention of our previous results (Yoshida and Hamano, 1993). Fig. 2 shows the structure of the boundary layers. It is obtained by subtracting the thermal wind solution and the geostrophic flow due to the Ekman suction from the numerical solution.

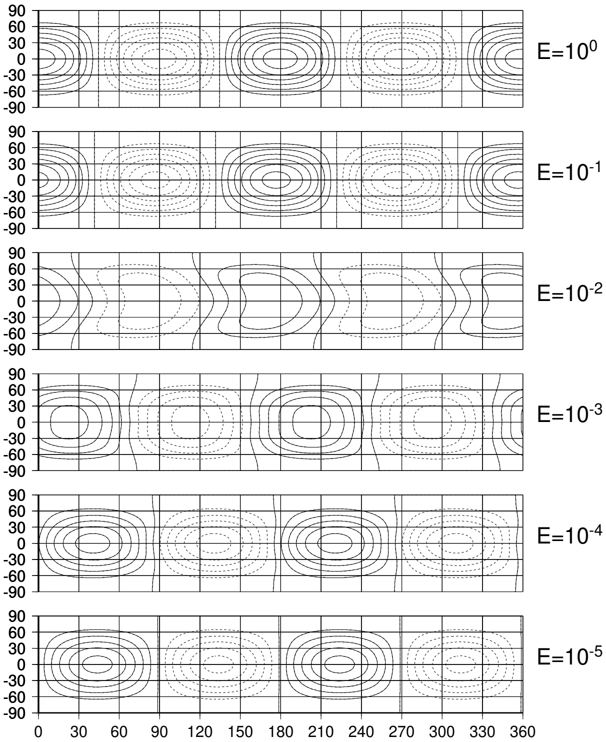

Fig. 1. Distributions of the radial velocity at $r = 0.7$ for the $(l, m) = (2, 2)$ CMB temperature pattern. The Ekman numbers are 10^0, 10^{-1},..., 10^{-5} from top to bottom. The abscissa is the longitude, and the ordinate is the latitude.

5 Reference

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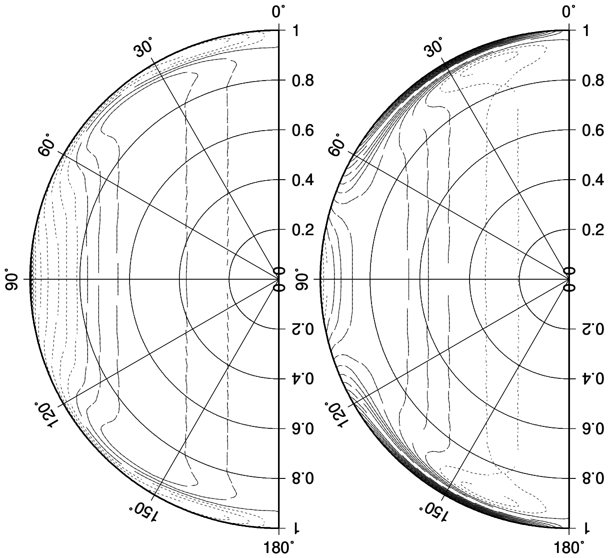

Fig. 2. Structure of the boundary layers for the $(l, m) = (2, 2)$ CMB temperature distribution for the Ekman number of 10^{-3}. In-phase and out-of-phase components of the cylindrically radial velocity are shown on the meridional plane.

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