Continuous excitation of Earth's free oscillations

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Abstract. Recently it has been evident that Earth's free oscillations are continuously excited at nano gal level. The atmospheric excitation hypothesis proposed by Kobayashi (1996) is suitable for the cause so far. We briefly review the recent progress in the study on the continuous free oscillations, discuss about the validity in the hypothesis and show open questions.

1 Introduction

The Sun pulsates due to acoustic waves generated by intensive turbulent gaseous motion on the top of the convective layer (e.g. Goldreich et al. 1994). Similar processes should occur on solid planets having atmospheres, and should excite seismic waves that propagate throughout the solid interior, thereby inducing free oscillations. Such planetary atmospheric disturbances should occur even on tectonically inactive solid planets such as Mars and Venus.

In the context, Kobayashi (1996) proposed atmospheric excitation hypothesis with positive suggestion for detection of such free oscillations. Then, Nawa et al. (1998) had tried to detect continuously excited free oscillations using a super conducting gravimeter at Showa station, Antarctica and they reported the first discovery of the phenomena. However their data are somewhat noisier to make the conclusion questionable. To support their conclusion, Suda et al. (1998), Tanimoto et al. (1998) and Kobayashi and Nishida (1998) reported continuous excitation of earth's free oscillations using other data sets from IDA, GEOSCOPE and IRIS stations. They all showed that such oscillations cannot be excited by earthquakes.

Here we summarize the recent results from observation which supports the atmospheric excitation hypothesis, and we review the atmospheric excitation mechanism proposed by Kobayashi (1996) and Kobayashi and Nishida (1998). In the latter issue, we especially focus on a discussion on the effective correlation length of atmospheric disturbances which is an important parameter for excitation amplitudes, and give a correction to the formulation of Kobayashi and Nishida (1998).

2 Results from observations

It had taken about 40 years to recognize that earth's background free oscillations are excited continuously, since Benioff et al. (1959) claimed that such oscillations could be excited by atmospheric disturbances. Nawa et al. (1998) presented the first observational evidence for the continuous excitation of Earth's free oscillations, and then many researchers have shown more attractive and reliable results (e.g. Suda et al. 1998, Nishida and Kobayashi 1999, Nishida et al. 2000, Roult and Crawford 2000) on the issue. They are summarized as follows.

1. Fundamental spheroidal modes in the mid mHz range (2.0–7.5 mHz) are continuously excited about 0.5 nano gal even in the least seismically active periods in the last two decades. The amplitudes are sufficiently large so that they cannot be attributed to earthquakes.

2. The amplitudes are about 0.5 nano gal and have a weak positive frequency dependence (like the square root of frequency). Below 2 mHz, only a few peaks can be identified since effects of Newton attraction of air mass fluctuations passing just above the sensor mask the peaks of free oscillations. Above 8 mHz, no clear peaks of free oscillations are observed, and instead, a broadened plateau of power spectral densities (PSDs) is seen around 10 mHz, then the PSDs decay and take the minimum value at about 15 mHz (Fig. 1).

3. ${}_0S_{29}$ and ${}_0S_{37}$ show somewhat larger amplitudes than those of around modes. The modes are located at branch crossings between (solid

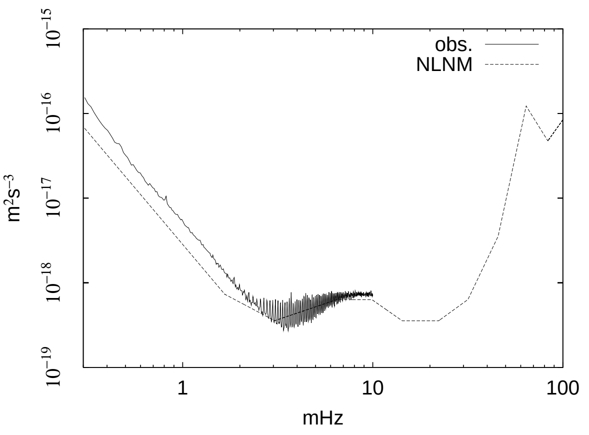

Fig. 1. Observed average power spectral densities on seismically quiet days and New Low Noise model by Peterson (1993). In the mid mHz band, continuously excited fundamental free oscillation modes are observed.

earth) surface waves and atmospheric sound waves.

4. The amplitudes show about 10 % annual variation in phase with the variation in the outward infrared flux from the Earth (larger in the summer season). The infrared flux can be thought to behave as a indicator of strength of convective motions in the atmosphere since the atmosphere is opaque for the infrared rays and energy must be carried outward by the advection. The annual variation of the strongly coupled mode ${}_0S_{29}$ is striking and up to about 40 % of its average amplitude.

5. Based on the observed PSD of atmospheric pressure $[2 \times 10^3 \times (f/f_0)^{-2}$ Pa^2/Hz], Fukao et al. (2001) obtained very weak frequency dependence of correlation length $[1 \times (f/f_0)^{-0.15}$ km] for the atmospheric pressure fluctuations to explain these free oscillation amplitudes, where $f_0 = 1$ mHz. These values are comparable with pressure fluctuation inferred from convective motion of the atmosphere responsible to the infrared flux as for size.

The above features of the oscillations qualitatively or semi-quantitatively support the hypothesis that these oscillations are excited by random action of the atmosphere on the ground. To reveal the nature of the continuous excitation more clearly, Nishida et al. (2000) successfully obtained a k–ω diagram (a spectral density contour plot on the plane of frequency and wave number) for the continuous background earth's free oscillations based on the random excitation. They calculated cross-spectra for any pair of selected 49 quiet stations, aligned them with respect to angular distance between stations and then convert the distance–frequency spectra to the k–ω diagram using Legendre functions. Since the diagram resolves ground motion spectra in not only frequency but also wave number (angular degree), we can stripe the free oscillation branches from other background noises. The diagram clearly shows the fundamental spheroidal oscillation branch from 1 mHz up to 20 mHz ($l \sim 200$) and very weak first and second overtone branches at the lower l and frequency area. The stripped normal mode amplitudes along the fundamental mode branch also have the plateau around 10 mHz and the minimum at 15 mHz. Thus the plateau seen in the PSD plot surely consists of the fundamental spheroidal mode oscillation energy although each modal peaks are not resolved(Fig. 1).

3 Atmospheric excitation

Formulation for the atmospheric excitation of earth's free oscillations have been developed by Kobayashi and Nishida (1998), Tanimoto and Um (1999) and Fukao et al. (2001). Acceleration amplitude of a normal mode is written in an intuitive form as

$$M_l \times a_l \sim P \times L^2 \times \sqrt{N} \times \sqrt{Q} \times \sqrt{2l+1}, \quad (1)$$

where M_l and a_l are modal mass and acceleration of the mode l respectively, P is intensity of pressure fluctuation at the normal mode frequency f_l which is the square root of the power contained in $f \geq f_l$, L is correlation length at the frequency, $N = 4\pi R^2/L^2$ is the number of independent forces, Q and l is the quality factor and the angular order of the mode respectively. $P \times L^2$ represents the expected intensity of the force in the coherent area L^2 for each cycle. Such forces stir the ground randomly and independently at different areas (in space) and different cycles (in time). So amplitude of each singlet is proportional to not $N \times Q$ but $\sqrt{N} \times \sqrt{Q}$. Note the singlet forget excitation before Q cycle ago, and $2l + 1$ singlets forming the multiplet are excited independently. Therefore, it is natural that the acceleration amplitude takes the form given by Eq. (1).

As mentioned in the previous section, heats converted from the visible lights from the sun by absorption on and near the surface are carried upward by convective motions of the atmosphere (Fig. 2). The heat transport equations are given by

$$\rho \frac{v_H^2}{H} = \rho \frac{\Delta T}{T} g, \quad (2)$$

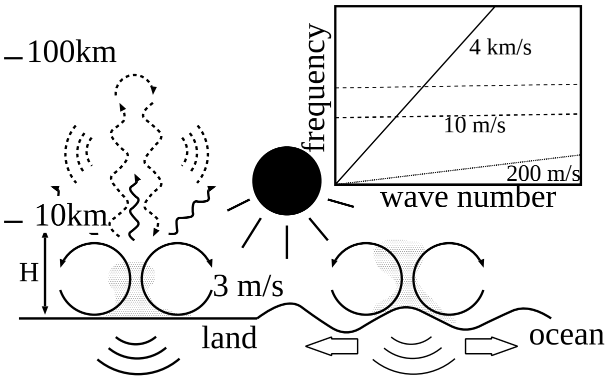

Fig. 2. Schematic view of excitation of elastic surface waves, ocean and atmospheric waves by atmospheric turbulent motions. Inset shows dispersion relations of those waves; surface waves (solid line; 4 km/s), ocean waves (dotted line; 200 m/s) and accoustic waves (broken line; 10 m/s).

$$\rho c_p v_H \Delta T \quad = \quad \frac{S(1-A)}{4}, \tag{3}$$

where ρ, T, H and c_p is the surface density, the surface temperature, the pressure scale height and the heat capacity at constant pressure of the atmosphere respectively, and v_H is the velocity of the convective motion, S is the solar radiation flux and A is the albedo (Kobayashi and Nishida 1998). By solving the equations, we obtained

$$v_H = \left(\frac{gHS(1-A)}{4\rho c_p T}\right)^{1/3}. \tag{4}$$

v_H is about 3.8 m/s for the appropriate values of the Earth's atmosphere. The corresponding pressure disturbance ($P_H = \rho v_H^2$) is then about 17 Pa and the mixing time ($\tau_H = H/v_H$) is about 2400 sec. Note v_H and P_H give their upper limits because in eqs. (2) and (3) we assume heats are transported by the convection with 100 % efficiency. However $v_H \sim 1$ m/s is so large that it gives very large Reynolds number Re$= Hv_H/\nu \sim 8 \times 10^8$ for the molecular viscosity $\nu \sim l \times c = 1 \times 10^{-5}$ m^2/s (l is the length of the mean free path of molecules and c is the sound velocity). So the convective motions are reasonably thought to be turbulent.

In the homogeneous and isotropic turbulence, there should be the inertial subrange somewhere between energy-containing eddy scale ($\sim H$ in our case) and the Kolmogorov scale at which molecular dissipation become important ($\sim 6 \times 10^{-4}$ m for the input energy rate and the molecular viscosity). In the inertial subrange, Kolmogorov (1941) obtained the energy spectrum function $E(\kappa, t) = \alpha\epsilon^{2/3}\kappa^{-5/3}$ by dimensional consideration, where α is an absolute constant, ϵ is input energy rate per unit mass and κ is wave number. If we adopt the power law, then $v \sim \sqrt{\int_\kappa^\infty E(\kappa', t) d\kappa'} \sim (\epsilon/\kappa)^{1/3}$. Since $c_p T/g = 7H/2$ for di-atomic molecular ideal gas, Eq. (4) can be written as $v_H = (\epsilon H)^{1/3}$, where

$$\epsilon = \frac{2}{7}\frac{S(1-A)}{4}\frac{1}{\rho H}.$$

By comparing above equations, we obtain

$$v = (\epsilon H)^{1/3}(H\kappa)^{-1/3} = v_H (H\kappa)^{-1/3},$$

and, therefore,

$$P \sim \rho v^2 = P_H (H\kappa)^{-2/3}.$$

Following the Taylor's hypothesis, this means

$$P = P_H \left(\frac{H}{v_H} v_H \kappa\right)^{-2/3} = P_H \left(\tau_H f\right)^{-2/3}, \tag{5}$$

where v_H is treated as the mean flow speed. Therefore the PSD of the pressure fluctuation is proportional to $f^{-7/3}$. On the other hand, the observed PSD of atmospheric pressure is proportional to f^{-2}.

The life time of a eddy with size $\lambda = 1/\kappa$ is $\tau' \sim \lambda^2/\nu_{\mathrm{e}} = \lambda/v$, where $\nu_{\mathrm{e}} \sim \lambda v$ is the eddy viscosity. Since the eddy is transported by the mean flow v_H during its life time τ', the eddy moves over the length $v_H\tau'$ in the flow direction. Directions of the mean flow v_H are different for different convective cells whose sizes are about H and whose life times are about H/v_H. Statistically speaking, the direction of the mean flow is isotropically distributed. Hence the mean length in a particular direction must be the diffusion length by v_H which is given by $L \sim \sqrt{Hv_H\tau'}$. Since $\lambda = v_H/f$, the length can be written as

$$L \sim \sqrt{Hv_H \frac{v_H}{vf}} = H(\tau_H f)^{-1/3}. \tag{6}$$

Therefore the correlation length at the frequency f is proportional to $f^{-1/3}$, which is comparable with the observed frequency dependence $f^{-0.15}$ by Fukao et al. (2001).

The following table is the summary comparing the observed and predicted pressure disturbances.

Table. 1. pressure intensity and correlation length

	observation	prediction
P	$1.4 \times (f/f_0)^{-0.5}$	$P_H(\tau_H f)^{-2/3}$
L	$1 \times (f/f_0)^{-0.15}$	$H(\tau_H f)^{-1/3}$

($f_0 = 1$ mHz, $P_H = 17$ Pa, $\tau_H = 2400$ sec; $H = 8.4$ km)

Although these values are comparable between the observation and the prediction, the predicted values are somewhat larger than the observed values and have slightly stronger frequency dependences. Using the predicted values, acceleration amplitudes of fundamental free oscillations near 4 mHz are about 1.8 nano gal which is about 4 times larger than the observed amplitudes. If we replace H by $H/4$, predicted amplitudes become the observational level.

In table 2, we summarize amplitudes of free oscillation modes for the three planets excited by atmospheric disturbances assuming our mechanism is valid on those planets. They have accidentally almost same values in spite of very different atmospheres.

Table. 2. Comparison between solid planets

planet	P_H	H	v_H	R	M	a
venus	104	15	0.4	6032	4.87	1.1
earth	17	8.5	3.8	6378	5.97	1.8
mars	2.5	11	13	3397	0.64	1.3
	Pa	km	m/s	km	10^{24} kg	n gal

Amplitudes are evaluated at 4 mHz assuming $Q = 200$ and surface wave velocity is 4 km/s for the three planets.

4 Discussion

As discussed in the previous section, the atmospheric random excitation theory successfully explains the observed amplitudes of free oscillations in the mid mHz band overall. However the trend of the predicted amplitudes gives larger amplitudes at and above 10 mHz. The spectral plateau around 10 mHz and the minimum near 15 mHz cannot be explained by the monotonous frequency dependence in the pressure fluctuations. To explain the spectral profile, we must adopt more strong frequency dependence in the forcing term above 8 mHz. It is surprising that the observed pressure PSD has f^{-2} dependence even above 10 mHz. There is no clear change in the frequency dependence. The discrepancy might be due to effect of oceans which soften the atmospheric forcing on the ocean floors above 10 mHz (although ocean waves are less effective for the excitation of the background free oscillations). In the present state, we have little information on the atmospheric disturbances, especially about correlation length of the disturbances. So we have a plan of an experiment to reveal atmospheric motions in the free oscillation band using an array of micro barometers. In the plan, we will also search atmospheric sound waves trapped in the middle atmosphere.

Acknowledgement. We thank Prof. H. Kawakatsu and the other committee members for holding the OHP/ION joint symposium, and we also thank all people developing and maintaining seismic stations for providing us high quality ground motion records.

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