Wave Solutions in Vertically Varying Media

•Spherical & plane wave solutions considered thus far are valid only within homogeneous space, or smoothly varying media.

•For vertically inhomogeneous media new solutions are found by solving boundary value problems, where there is continuity of traction and displacement across media discontinuities. i.e. stress & displacement are transmitted and surfaces remain in contact.

•We consider reflection and refraction of SH waves at a welded discontinuity.

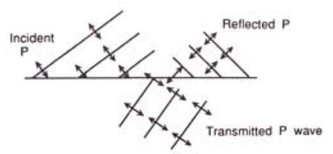
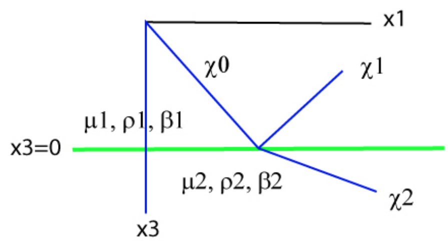
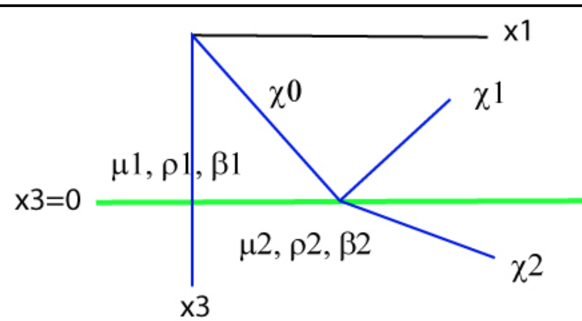


FIGURE 3.24 *P*-wave particle motions for the incident, reflected, and refracted *P* waves. Note that if this is a solid-solid boundary, the shear stress in the two layers will *not* match at the boundary, requiring the generation of *SV* motion in both media.





$$\chi_0(\vec{x},t) = A \cdot e^{i(\omega t - \omega p x_1 - \omega \eta_1 x_{31})}$$

$$\chi_1(\vec{x},t) = B \cdot e^{i(\omega t - \omega p x_1 + \omega \eta_1 x_{31})}$$

$$\chi_2(\vec{x},t) = C \cdot e^{i(\omega t - \omega p x_1 - \omega \eta_2 x_{31})}$$

System of Equations

$$\bar{u} = \nabla\phi + \nabla\times\langle 0,0,\chi\rangle = -\frac{\partial\chi}{\partial x_1}\hat{x}_2$$ Helmholtz Equation

$$\sigma_{ij} = \lambda\theta\delta_{ij} + 2\mu\varepsilon_{ij} \qquad \sigma_{32} = \mu\frac{\partial u_2}{\partial x_3} = \mu\frac{\partial^2\chi}{\partial x_1\partial x_3}$$ Constitutive Equation

$$\left(-\frac{\partial\chi_0}{\partial x_1}\right)_{x_3=0} + \left(-\frac{\partial\chi_1}{\partial x_1}\right)_{x_3=0} = \left(-\frac{\partial\chi_2}{\partial x_1}\right)_{x_3=0}$$

First consider continuity of displacement at x_3=0 boundary

$$i\omega pAe^{i(\omega t-\omega px)} + i\omega pBe^{i(\omega t-\omega px)} = i\omega pCe^{i(\omega t-\omega px)}$$

$$A + B = C$$

$$\left(-\mu_1\frac{\partial^2\chi_0}{\partial x_1\partial x_3}\right)_{x_3=0} + \left(-\mu_1\frac{\partial^2\chi_1}{\partial x_1\partial x_3}\right)_{x_3=0} = \left(-\mu_2\frac{\partial^2\chi_2}{\partial x_1\partial x_3}\right)_{x_3=0}$$

Next, consider continuity of traction at x_3=0

$$-(-i\omega p)(-i\omega\eta_1\mu_1)Ae^{i(\omega t-\omega px_1)} - (-i\omega p)(i\omega\eta_1\mu_1)Be^{i(\omega t-\omega px_1)} = -(-i\omega p)(-i\omega\eta_2\mu_2)Ce^{i(\omega t-\omega px_1)}$$

$$A\mu_1\eta_1 - B\mu_1\eta_1 = C\mu_2\eta_2$$

$$A + B = C$$ Equation 1

$$A\mu_1\eta_1 - B\mu_1\eta_1 = C\mu_2\eta_2$$ Equation 2

First solve: $(\mu_1\eta_1)(1) + (2)$

$$\begin{aligned}
&\mu_1\eta_1 A + \mu_1\eta_1 B = \mu_1\eta_1 C \\
&+ \\
&\mu_1\eta_1 A - \mu_1\eta_1 B = \mu_2\eta_2 C \\
\hline
&2\mu_1\eta_1 A = (\mu_1\eta_1 + \mu_2\eta_2)C
\end{aligned}$$

$$T = \frac{C}{A} = \frac{2\mu_1\eta_1}{(\mu_1\eta_1 + \mu_2\eta_2)}$$

Next solve: $(-\mu_2\eta_2)(1) + (2)$

$$\begin{aligned}
&-\mu_2\eta_2 A - \mu_2\eta_2 B = -\mu_2\eta_2 C \\
&+ \\
&\mu_1\eta_1 A - \mu_1\eta_1 B = \mu_2\eta_2 C \\
\hline
&(\mu_1\eta_1 - \mu_2\eta_2)A - (\mu_1\eta_1 + \mu_2\eta_2)B = 0
\end{aligned}$$

$$R = \frac{B}{A} = \frac{(\mu_1\eta_1 - \mu_2\eta_2)}{(\mu_1\eta_1 + \mu_2\eta_2)}$$

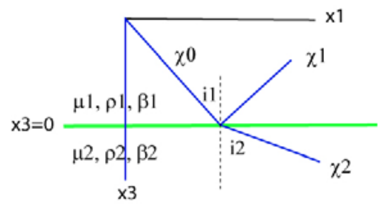
Since η is a function of p (horizontal slowness), $\eta_j = \frac{\cos(i)}{\beta_j} = \sqrt{\frac{1}{\beta_j^2} - p^2}$, the reflection (R), and transmission (T) coefficients depend on the angle of incidence of the wave with respect to the media discontinuity

also remember that $\mu_1 = \beta_1^2 \rho_1$

$$T = \frac{2\beta_1\rho_1\cos(i_1)}{\left(\beta_1\rho_1\cos(i_1) + \beta_2\rho_2\cos(i_2)\right)} \qquad R = \frac{\left(\beta_1\rho_1\cos(i_1) - \beta_2\rho_2\cos(i_2)\right)}{\left(\beta_1\rho_1\cos(i_1) + \beta_2\rho_2\cos(i_2)\right)}$$

Normal Incidence

$$T = \frac{2\beta_1\rho_1}{\left(\beta_1\rho_1 + \beta_2\rho_2\right)} \qquad R = \frac{\left(\beta_1\rho_1 - \beta_2\rho_2\right)}{\left(\beta_1\rho_1 + \beta_2\rho_2\right)}$$

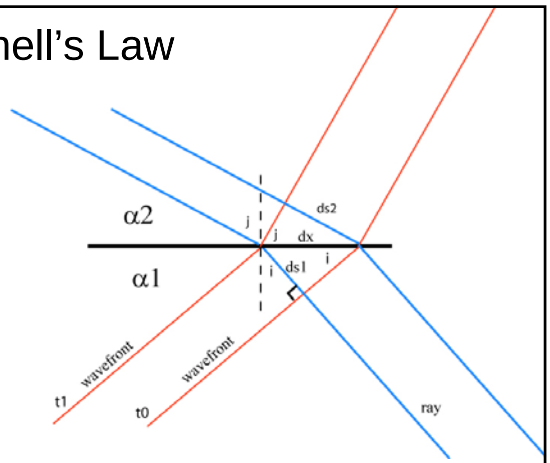


If $i_1 = i_c$ then $\eta_2 = 0$ and R=1 and T=2

If $i_1 > i_c$ we have wide-angle (total internal) reflection
T=0
R becomes complex

$$if\ \ i_1 > i_c\ \ then \left(\frac{1}{\beta_2^2} - p^2\right) < 0 \quad \eta_2 \rightarrow \tilde{\eta}_2 = \pm i\sqrt{p^2 - \frac{1}{\beta_2^2}}$$

Snell's Law



$$dt = \frac{ds_1}{\alpha_1} = \frac{dx \cdot \sin(i)}{\alpha_1} = \frac{dx \cdot \sin(j)}{\alpha_2}$$

$$\frac{dt}{dx} = \frac{\sin(i)}{\alpha_1} = \frac{\sin(j)}{\alpha_2} = p$$

Snell's Law is General

$$\frac{dt}{dx} = p = \frac{\sin(i)}{\alpha_1} = \frac{\sin(j)}{\alpha_2} = \frac{\sin(k)}{\beta_2} = etc.$$

If $\alpha_2 > \alpha_1$ $\quad j > i$

If $\alpha_2 < \alpha_1$ $\quad j < i$

Behavior of $R_{SH}(p)$ for $p \geq 1/\beta_2$, $i > i_c$

$$R(p)\Big|_{p \geq 1/\beta_2} = \frac{\mu_1 \eta_1 - i\mu_2 \tilde{\eta}_2}{\mu_1 \eta_1 + i\mu_2 \tilde{\eta}_2}, \quad \tilde{\eta}_2 = \left[p^2 - 1/\beta_2^2\right]^{1/2}$$

$R(p)$ can ∴ be written in terms of its amplitude and phase, where

$$\|R(p)\| = \frac{\left[\mu_1^2 \eta_1^2 + \mu_2^2 \tilde{\eta}_2^2\right]^{1/2}}{\left[\mu_1^2 \eta_1^2 + \mu_2^2 \tilde{\eta}_2^2\right]^{1/2}} = 1$$

Phase, $\Phi = 2 \cdot \tan^{-1}\left(\frac{\mu_2 \tilde{\eta}_2}{\mu_1 \eta_1}\right)$

$\therefore \quad R(p)\Big|_{p \geq 1/\beta_2} = e^{i\phi}$

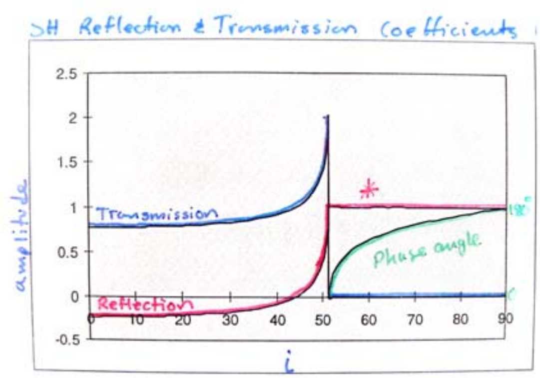
and $\quad x_2 = 1 \cdot e^{i\omega(t - px_1 - \eta_1 x_3 + \frac{\phi}{\omega})}$

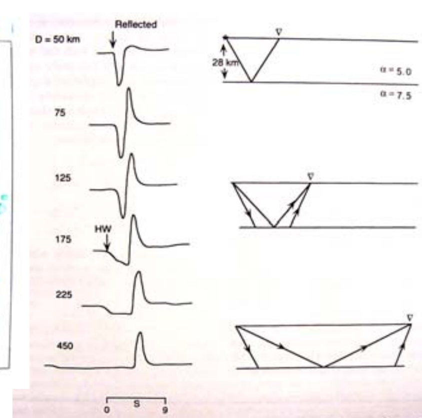
Thus for a post-critically reflected wave $\|R(p)\| = 1$ indicating that the wave is totally reflected however a phase shift is introduced which produces earlier arrivals since

$t - px_1 - \eta_1 x_3 + \frac{\phi}{\omega} = c \qquad t = px_1 + \eta_1 x_3 - \frac{\phi}{\omega}$

assuming $c = 0$

SH R&T PLOT





$$T_{SH}(p)\Big|_{p \geq 1/\beta_2} = \left(\frac{2\mu_1 n_1}{\mu_1 n_1 + i\mu_2 \tilde{n}_2}\right) \cdot \left(\frac{\mu_1 n_1 - i\mu_2 \tilde{n}_2}{\mu_1 n_1 - i\mu_2 \tilde{n}_2}\right)$$

$$= \frac{2\mu_1^2 n_1^2}{\mu_1^2 n_1^2 + \mu_2^2 \tilde{n}_2^2} - \frac{2i\mu_1 n_1 \mu_2 \tilde{n}_2}{\mu_1^2 n_1^2 + \mu_2^2 \tilde{n}_2^2}$$

For transmitted waves $p \geq 1/\beta_2$ P exists only at $p = 1/\beta_2$

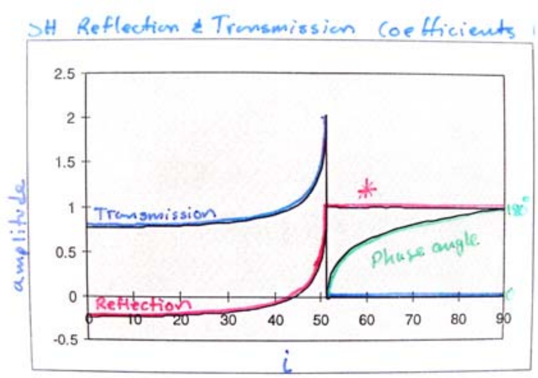
thus

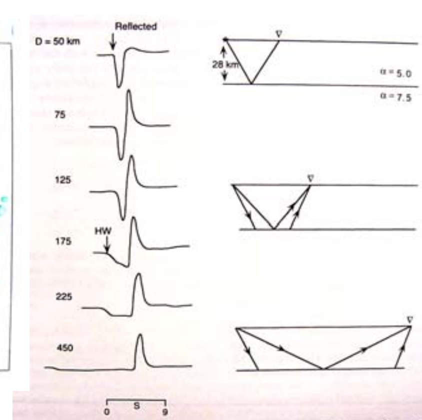
$$\chi_2(\bar{x}, t) = T(p)\, e^{i\omega(t - p x_1 \pm i\tilde{n}_2 x_3)}$$

$$= T(p)\, e^{-\omega \tilde{n}_2 |x_3|} \cdot e^{i\omega(t - p x_1)}$$

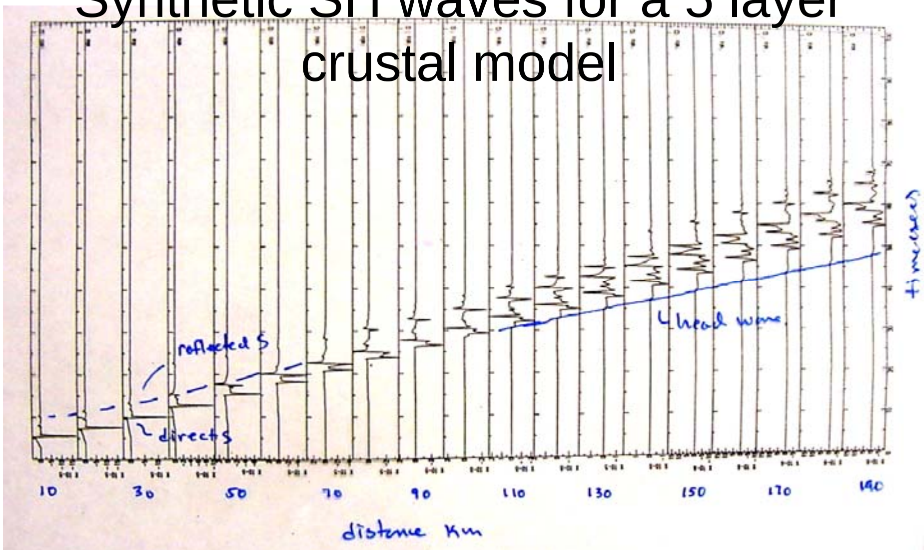
$$= 2 e^{i\omega(t - x_1/\beta_2)} \qquad x_3 = \emptyset$$

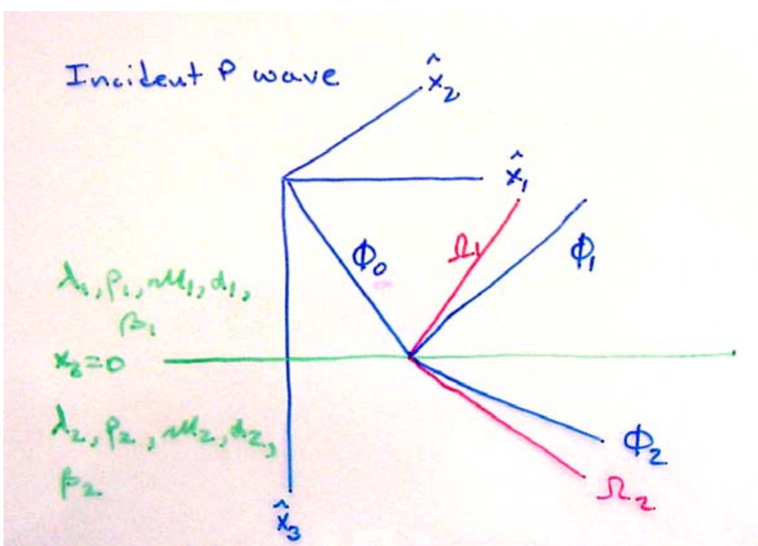
SH R&T PLOT





Synthetic SH waves for a 5 layer crustal model





$$\phi_0 = A e^{i\omega(t - p x_1 - \eta_{\alpha_1} x_3)}$$

$$\phi_1 = B e^{i\omega(t - p x_1 + \eta_{\alpha_1} x_3)} \qquad \Omega_1 = D e^{i\omega(t - p x_1 + \eta_{\beta_1} x_3)}$$

$$\phi_2 = C e^{i\omega(t - p x_1 - \eta_{\alpha_2} x_3)} \qquad \Omega_2 = E e^{i\omega(t - p x_1 - \eta_{\beta_2} x_3)}$$

$$\sigma_{31} = \mu\left(\frac{\partial u_3}{\partial x_1} + \frac{\partial u_1}{\partial x_3}\right)$$

$$\sigma_{33} = \lambda \Theta + 2\mu \frac{\partial u_3}{\partial x_3}$$

$$\vec{u} = \left\langle \frac{\partial \Phi}{\partial x_1}, \frac{\partial \Phi}{\partial x_2}, \frac{\partial \Phi}{\partial x_3} \right\rangle + \nabla \times \langle 0, \Omega, 0 \rangle$$

$$= \left(\frac{\partial \Phi}{\partial x_1} - \frac{\partial \Omega}{\partial x_3}\right)\hat{x}_1 + \left(\frac{\partial \Phi}{\partial x_3} + \frac{\partial \Omega}{\partial x_1}\right)\hat{x}_3$$

Solve boundary value problem for continuity of stress across $\hat{x}_3$ surface (σ_{31}, σ_{33}) and displacement in $\hat{x}_1\hat{x}_3$ plane.

TABLE 3.1 Displacement Reflection and Transmission Coefficients

Coefficient	Formula
Solid–free surface (P–SV)	
R_{PP}	$\{-[(1/\beta^2) - 2p^2]^2 + 4p^2\eta_\alpha\eta_\beta\}/A$
R_{PS}	$\{4(\alpha/\beta)p\eta_\alpha[(1/\beta^2) - 2p^2]\}/A$
R_{SP}	$\{4(\beta/\alpha)p\eta_\beta[(1/\beta^2) - 2p^2]\}/A$
R_{SS}	$\{-[(1/\beta^2) - 2p^2]^2 + 4p^2\eta_\alpha\eta_\beta\}/A$
$R_{SS}(SH)$	1
Solid–solid (P–SV)	
R_{PP}	$[(b\eta_{\alpha_1} - c\eta_{\alpha_2})F - (a + d\eta_{\alpha_1}\eta_{\beta_2})Hp^2]/D$
R_{PS}	$-[2\eta_{\alpha_1}(ab + cd\eta_{\alpha_2}\eta_{\beta_2})p(\alpha_1/\beta_1)]/D$
T_{PP}	$[2\rho_1\eta_{\alpha_1}F(\alpha_1/\alpha_2)]/D$
T_{PS}	$[2\rho_1\eta_{\alpha_1}Hp(\alpha_1/\beta_2)]/D$
R_{SS}	$-[(b\eta_{\beta_1} - c\eta_{\beta_2})E - (a + b\eta_{\alpha_2}\eta_{\beta_1})Gp^2]/D$
R_{SP}	$-[2\eta_{\beta_1}(ab + cd\eta_{\alpha_2}\eta_{\beta_2})p(\beta_1/\alpha_1)]/D$
$R_{SS}(SH)$	$\dfrac{\mu_1\eta_{\beta_1} - \mu_2\eta_{\beta_2}}{\mu_1\eta_{\beta_1} + \mu_2\eta_{\beta_2}}$
$T_{SS}(SH)$	$\dfrac{2\mu_1\eta_{\beta_1}}{\mu_1\eta_{\beta_1} + \mu_2\eta_{\beta_2}}$

$a = \rho_2(1 - 2\beta_2^2p^2) - \rho_1(1 - 2\beta_1^2p^2)$ $\qquad E = b\eta_{\alpha_1} + c\eta_{\alpha_2}$

$b = \rho_2(1 - 2\beta_2^2p^2) - 2\rho_1\beta_1^2p^2$ $\qquad F = b\eta_{\beta_1} + c\eta_{\beta_2}$

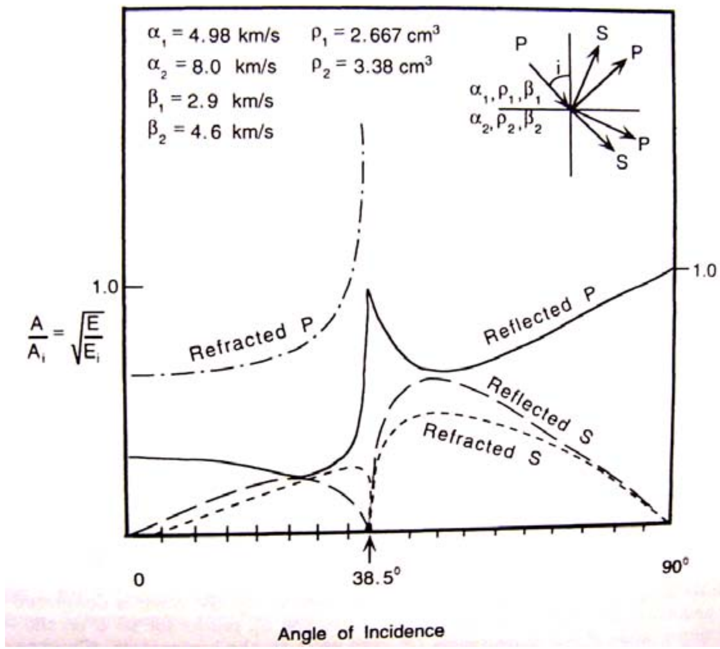
$c = \rho_1(1 - 2\beta_1^2p^2) + 2\rho_2\beta_2^2p^2$ $\qquad G = a - d\eta_{\alpha_1}\eta_{\beta_2}$

$d = 2(\rho_2\beta_2^2 - \rho_1\beta_1^2)$ $\qquad H = a - d\eta_{\alpha_2}\eta_{\beta_1}$

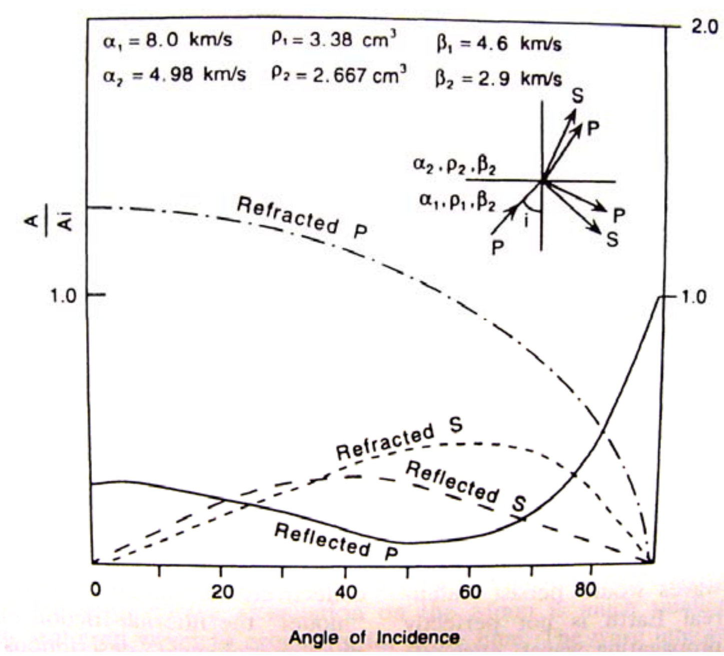
$$D = EF + GHp^2$$

$$A = [(1/\beta^2) - 2p^2]^2 + 4p^2\eta_{\alpha_1}\eta_{\beta_1}$$

Downward incident
What is the form for upward
Incident?



α1 = 4.98 km/s
α2 = 8.0 km/s
β1 = 2.9 km/s
β2 = 4.6 km/s
ρ1 = 2.667 cm3
ρ2 = 3.38 cm3
α1, ρ1, β1
α2, ρ2, β2
P
S
P
i
P
S
A/Ai = √(E/Ei)
1.0
1.0
Refracted P
Reflected P
Reflected S
Refracted S
0
38.5°
90°
Angle of Incidence



α1 = 8.0 km/s
α2 = 4.98 km/s
ρ1 = 3.38 cm3
ρ2 = 2.667 cm3
β1 = 4.6 km/s
β2 = 2.9 km/s
2.0
α2, ρ2, β2
α1, ρ1, β2
S
P
P
S
i
P
A/Ai
1.0
1.0
Refracted P
Refracted S
Reflected S
Reflected P
0
20
40
60
80
Angle of Incidence

Example – Northridge Earthquake

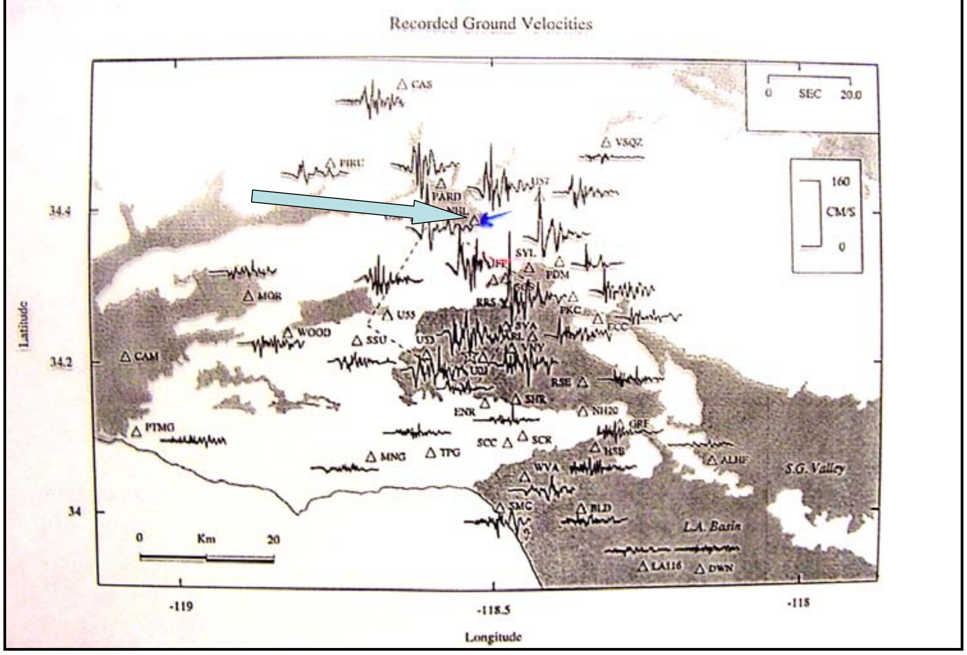


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Table 1
Northridge Regional Velocity Structure

Rock Model

A. Rock Stations

Vp (km/sec)	Vs (km/sec)	Density (g/cm)	Thickness (km)	Depth (km)
1.9	1.0	2.1	0.5	0.0
4.0	2.0	2.4	1.0	0.5
5.5	3.2	2.7	2.5	1.5
6.3	3.6	2.8	23.0	4.0
6.8	3.9	2.9	13.0	27.0
7.8	4.5	3.3		40.0

Soil Model

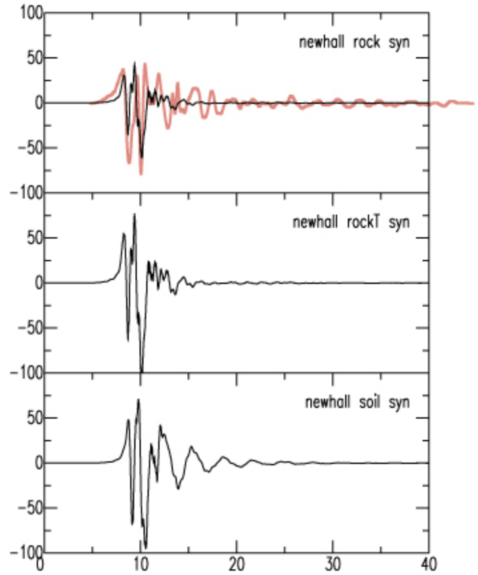
B. Soil Stations

Layer	Vp (km/sec)	Vs (km/sec)	Density (g/cm)	Thickness (km)	Depth (km)	
1	0.8	0.3	1.7	0.1	0.0	T_{21}
2	1.2	0.5	1.8	0.2	0.1	T_{32}
3	1.9	1.0	2.1	0.2	0.3	
4 (Rock)	4.0	2.0	2.4	1.0	0.5	
5 (Rock)	5.5	3.2	2.7	2.5	1.5	
6 (Rock)	6.3	3.6	2.8	23.0	4.0	
7 (Rock)	6.8	3.9	2.9	13.0	27.0	
8 (Rock)	7.8	4.5	3.3		40.0	

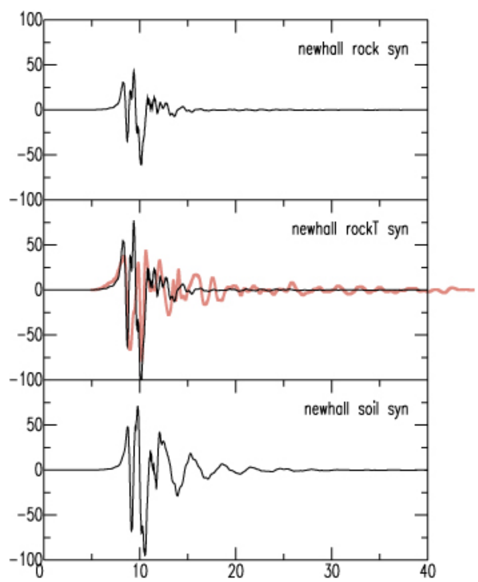
$T_{32} = 1.40$

$T_{21} = 1.27$

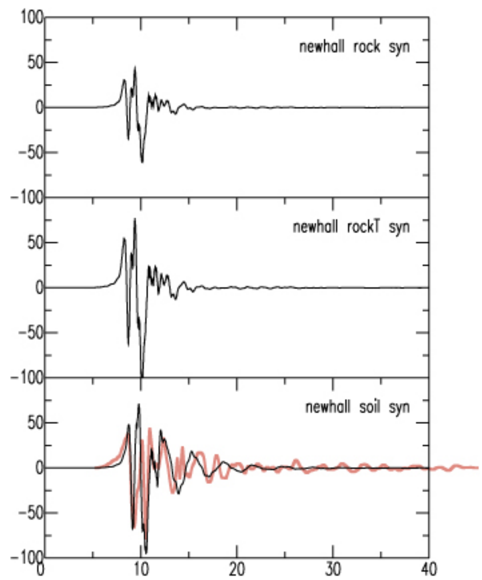
$T = T_{32} \cdot T_{21} = 1.78$



Rock Synthetic



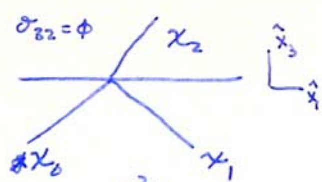
Scaled (x1.78) synthetic



Complete soil synthetic

R_{SH} @ free surface

~~σ_{32}~~



from the continuity of stress equation

$$\sigma_{32} = \frac{d^2\chi}{dx_1 dx_3}\mu$$

$$\mu_1 \frac{d^2\chi_0}{dx_1 dx_3}\Big|_{x_3=0} + \mu_1 \frac{d^2\chi_1}{dx_1 dx_3}\Big|_{x_3=0} = 0$$

$$\mu_1(-i\omega p)(-i\omega \eta_1)A + B\mu_1(-i\omega p)(i\omega \eta_1) = 0$$

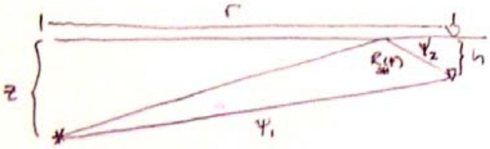
$$-A + B = 0$$

$$A = B$$

$R_{SH} = 1$ @ free surface

Free Surface Focussing

- consider the following geometry for an incident SH plane wave



- The total displacement potential is then

$$\Psi(\vec{x},t) = \Psi_1(\vec{x},t) + \Psi_2(\vec{x},t)$$

$$= A e^{i\omega(t - px_1 + \eta(z-h))} + A \cdot R_{SH} e^{i\omega(t - px_1 + \eta z - \eta h)}$$

Free surface.

$$\lim_{h\to 0} \Psi(\vec{x},t) = (A + A R_{SH,FS}) e^{i\omega(t - px_1 + \eta z)}$$

$$= \underline{\underline{2}} A e^{i\omega(t - px_1 + \eta z)}$$

Example: Richmond Field Station

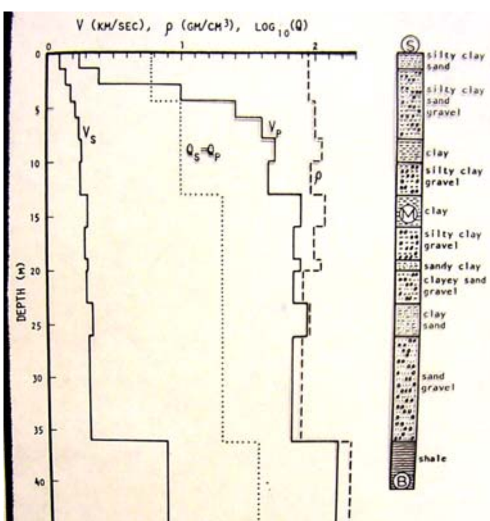


FIG. 2. The depth profiles of velocities, quality factors, and density which were estimated for the vicinity of the vertical seismic array. The section on the *right* was derived from drill logs and drill cores. The location of seismic sensors at the bottom of the hole, mid-hole, and surface are denoted by B, M, and S, respectively.

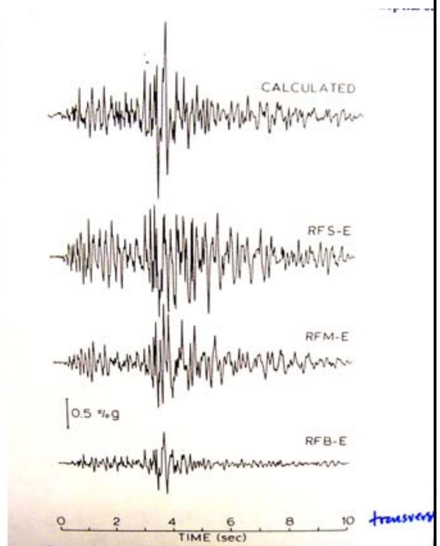


FIG. 4. Similar to Figure 3 for the northeast component of motion.

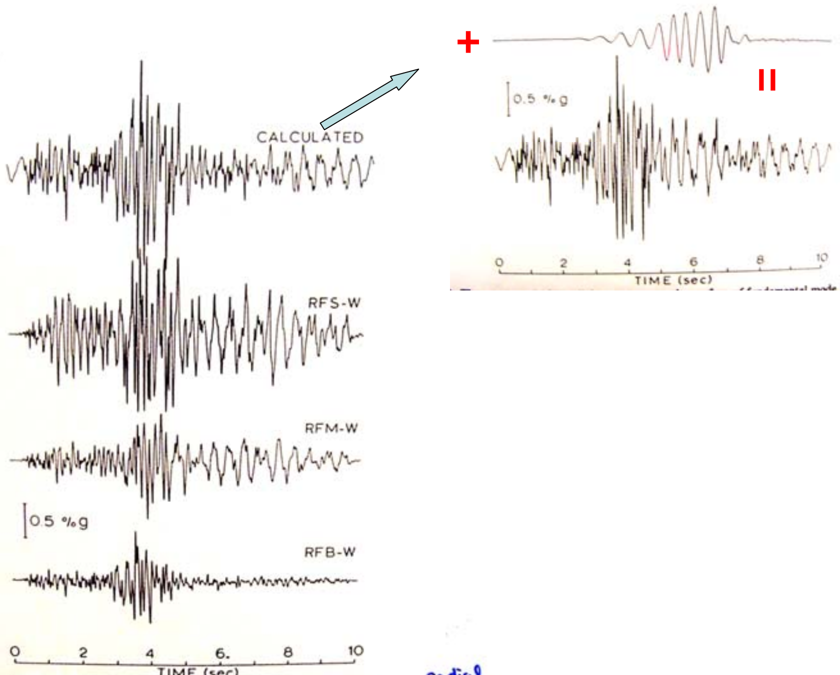
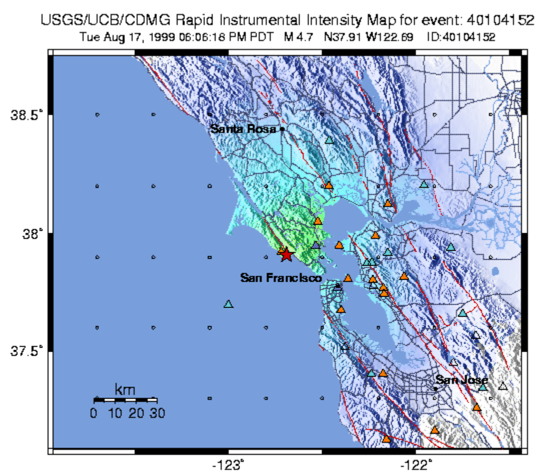
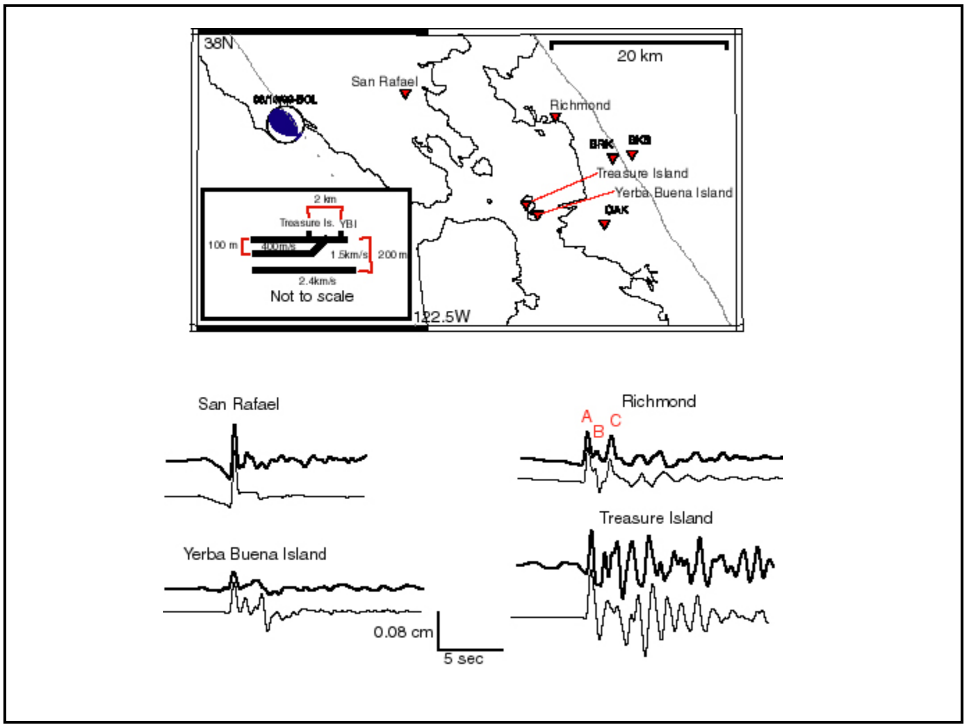


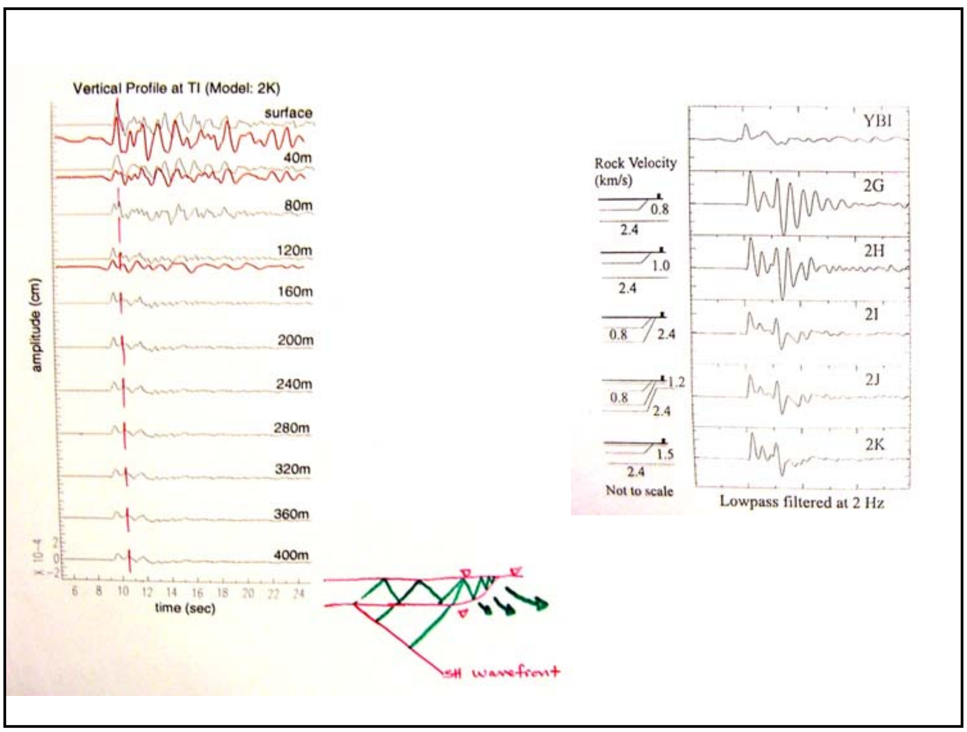
FIG. 5. Similar to Figure 3 for the northwest component of motion.

Example: Bolinas



PERCEIVED SHAKING	Not felt	Weak	Light	Moderate	Strong	Very strong	Severe	Violent	Extreme
POTENTIAL DAMAGE	none	none	none	Very light	Light	Moderate	Moderate/Heavy	Heavy	Very Heavy
PEAK ACC.(%g)	<.17	.17-1.4	1.4-3.9	3.9-9.2	9.2-18	18-34	34-65	65-124	>124
PEAK VEL.(cm/s)	<0.1	0.1-1.1	1.1-3.4	3.4-8.1	8.1-16	16-31	31-60	60-116	>116
INSTRUMENTAL INTENSITY	I	II-III	IV	V	VI	VII	VIII	IX	X+


38N
20 km
San Rafael
Richmond
BRK
BKS
Treasure Island
Yerba Buena Island
OAK
2 km
Treasure Is.
YBI
100 m
1.5km/s
200 m
2.4km/s
Not to scale
122.5W
San Rafael
Richmond
A
B
C
Treasure Island
Yerba Buena Island
0.08 cm
5 sec


Vertical Profile at TI (Model: 2K)
surface
40m
80m
120m
160m
200m
240m
280m
320m
360m
400m
amplitude (cm)
time (sec)
Rock Velocity (km/s)
Not to scale
YBI
2G
2H
2I
2J
2K
Lowpass filtered at 2 Hz
SH wavefront

Energy / Energy Flux should be conserved

$R^2 + T^2 \neq 1$ giving the appearance that energy is not conserved

The fact that energy is carried to and away from an interface at different angles needs to be considered

$$E = \left[\frac{1}{2}\rho\left(\frac{du}{dt}\right)^2 + \frac{1}{2}\mu\left(\frac{du}{dx}\right)^2\right]\delta A\,\delta x$$

$$E = \left[\frac{1}{2}\rho\left(\frac{du}{dt}\right)^2 + \frac{1}{2}\mu\left(\frac{du}{dx}\right)^2\right]\delta A\,\delta x$$

assuming for SH that $u = B\cos(\omega t - Kx)$

where $\frac{\omega}{K} = \beta$

$\frac{du}{dt} = -B\omega\sin(\omega t - Kx)$ & $\frac{du}{dx} = BK\sin(\omega t - Kx)$

$$\therefore \quad \frac{du}{dx} = -\frac{1}{\beta}\frac{du}{dt}$$

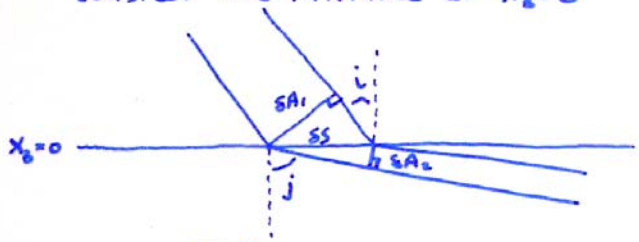
then

$$E = \rho\left(\frac{du}{dt}\right)^2 \delta A\,\delta x = \rho\beta\left(\frac{du}{dt}\right)^2 \delta A\,\delta t$$

the energy flux is then

$$I = \rho\beta\left(\frac{du}{dt}\right)^2 \qquad \frac{Kg}{m^3}\cdot\frac{m}{s}\cdot\frac{m^2}{s^2} \rightarrow \frac{(N.m)}{m^2.s}$$

Consider the interface at $x_3 = 0$



$$E = \rho_1 \beta_1 \left(\frac{\partial u}{\partial t}\right)^2 (\delta S \cos i)\, \delta t$$

$$= \rho_1 \beta_1 \frac{\beta \cos i}{\beta_1} B^2 \omega^2 \sin^2(\omega t - Kx)\, \delta t\, \delta S$$

$$= \mu_1 \eta_1 B^2 \omega^2 \sin^2(\omega t - Kx)\, \delta t\, \delta S$$

$$E_{inc.} = E_{refl.} + E_{refr.}$$

$$\mu_1 \eta_1 = \mu_1 \eta_1 R^2 + \mu_2 \eta_2 T^2$$

$$1 = R^2 + \frac{\mu_2 \eta_2}{\mu_1 \eta_1} T^2$$

Energy Partioning in SH Waves

$\rho 1 := 2000 \quad \rho 2 := 2670 \quad \beta 1 := 2.0 \quad \beta 2 := 3.5$ SI Units

$\mu 1 := \beta 1^2 \cdot \rho 1 \quad \mu 2 := \beta 2^2 \cdot \rho 2$

$\mu 1 = 8 \cdot 10^3 \quad \mu 2 = 3.271 \cdot 10^4$

$\eta 1(p) := \sqrt{\frac{1}{\beta 1^2} - p^2} \quad \eta 2(p) := \sqrt{\frac{1}{\beta 2^2} - p^2}$

Reflection & Transmitted Energy - Waves Incident from Above

$$R(p) := \frac{\mu 1 \cdot \eta 1(p) - \mu 2 \cdot \eta 2(p)}{\mu 1 \cdot \eta 1(p) + \mu 2 \cdot \eta 2(p)} \qquad T(p) := \begin{cases} \dfrac{2 \cdot \mu 1 \cdot \eta 1(p)}{\mu 1 \cdot \eta 1(p) + \mu 2 \cdot \eta 2(p)} & \text{if } p \le \dfrac{1}{\beta 2} \\ 0.0 & \text{otherwise} \end{cases}$$

Transmission Energy Coefficient $E(p) := \dfrac{\mu 2 \cdot \eta 2(p)}{\mu 1 \cdot \eta 1(p)}$ i.e. Ray area nomalization factor

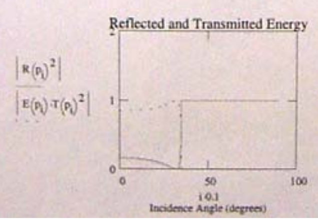
$N := 899$

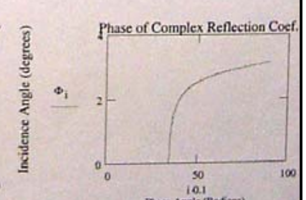
$i := 0..N$

$\theta_i := \dfrac{\pi}{180} \cdot i \cdot 0.1$

$p_i := \dfrac{\sin(\theta_i)}{\beta 1}$

$$\Phi_i := \begin{cases} 0 & \text{if } p_i < \dfrac{1}{\beta 2} \\ \arg(R(p_i)) - 1 & \text{otherwise} \end{cases}$$



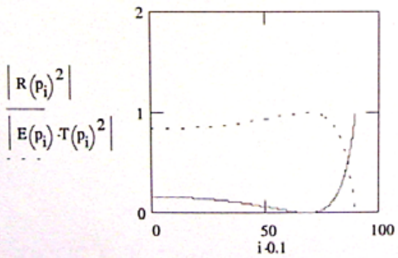


Reflection & Transmitted Energy - Waves Incident from Below

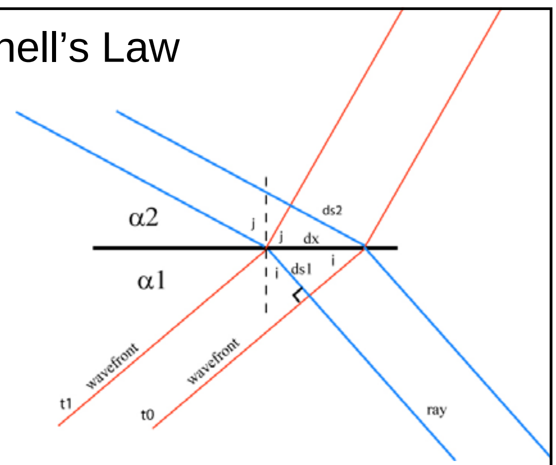
$$R(p) := \frac{(\mu 2 \cdot \eta 2(p)) - \mu 1 \cdot \eta 1(p)}{\mu 1 \cdot \eta 1(p) + \mu 2 \cdot \eta 2(p)} \qquad T(p) := \frac{2 \cdot \mu 2 \cdot \eta 2(p)}{\mu 1 \cdot \eta 1(p) + \mu 2 \cdot \eta 2(p)}$$

$$E(p) := \frac{\mu 1 \cdot \eta 1(p)}{\mu 2 \cdot \eta 2(p)}$$

$$p_i := \frac{\sin(\theta_i)}{\beta 2}$$



Snell's Law



$$dt = \frac{ds_1}{\alpha_1} = \frac{dx \cdot \sin(i)}{\alpha_1} = \frac{dx \cdot \sin(j)}{\alpha_2}$$

$$\frac{dt}{dx} = \frac{\sin(i)}{\alpha_1} = \frac{\sin(j)}{\alpha_2} = p$$

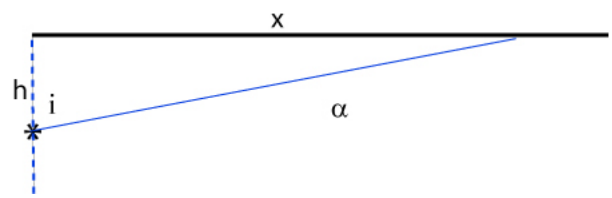
Snell's Law is General

$$\frac{dt}{dx} = p = \frac{\sin(i)}{\alpha_1} = \frac{\sin(j)}{\alpha_2} = \frac{\sin(k)}{\beta_2} = etc.$$

If $\alpha_2 > \alpha_1$ $\quad j > i$

If $\alpha_2 < \alpha_1$ $\quad j < i$

Direct Wave



$$\tau = px + \eta h$$

$$= \frac{x}{R\alpha}x + \frac{h}{R\alpha}h$$

$$= \frac{(x^2 + h^2)}{R\alpha} = \frac{\sqrt{x^2 + h^2}}{\alpha} = \frac{R}{\alpha}$$

Alternative Solution Technique

Suppose x, h, and α are given. How can we solve for p, arrival time (T) and the reflection transmission coefficients?

Shooting rays

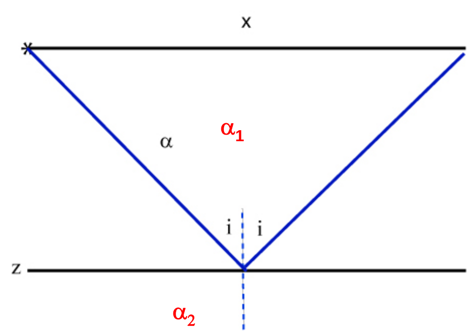
$$x_{given} = h \cdot \tan(i)$$ Find value of *i* that gives desired distance

$$p = \frac{\sin(i)}{\alpha}$$ Calculate the ray parameter

$$\tau = px + \eta h$$ Calculate arrival time and other parametes (e.g. transmission & reflection coefficients)

This seems more difficult, but there is a clear advantage in more complicated problems.

Reflection



$$\tau = px + 2\eta z$$

$$= \frac{x}{2R\alpha} x + 2\frac{z}{R\alpha} z$$

$$= \frac{x^2 + 4z^2}{2\alpha\sqrt{\frac{x^2}{4} + z^2}} = \frac{\sqrt{x^2 + 4z^2}}{\alpha}$$

$$\tau = \frac{2R}{\alpha} = \frac{2\sqrt{\frac{x^2}{4} + z^2}}{\alpha}$$

$$= \frac{\sqrt{x^2 + 4z^2}}{\alpha}$$

Alternative Method

$$x_{given} = 2h \cdot \tan(i)$$ Find value of *i* that gives desired distance

$$p = \frac{\sin(i)}{\alpha}$$ Calculate the ray parameter

$$\tau = px + 2\eta h$$ Calculate arrival time and other parametes (e.g. transmission & reflection coefficients)

Head wave

Arrival Time

$$T = px + 2\eta z$$

$$= \frac{x}{\alpha_2} + 2\left[\frac{1}{\alpha_1^2} - \frac{1}{\alpha_2^2}\right]^{1/2} z$$

Critical Distance

$$\tan(i) = \frac{p}{\eta} = \frac{x}{2z}$$

$$x_c = 2z\frac{p}{\eta} = 2z\frac{1}{\beta_2}\frac{1}{\left[\frac{1}{\beta_1^2} - \frac{1}{\beta_2^2}\right]^{1/2}}$$

$$= 2z\frac{\beta_1}{\sqrt{\beta_2^2 - \beta_1^2}}$$

Cross-over distance

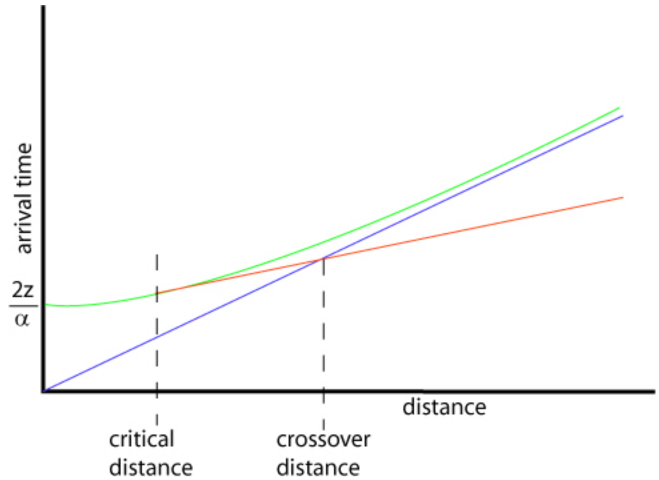
$$\frac{x}{\alpha_1} = \frac{x}{\alpha_2} + 2\left[\frac{1}{\alpha_1^2} - \frac{1}{\alpha_2^2}\right]^{1/2} z$$

$$\frac{x^2(\alpha_2 - \alpha_1)^2}{\alpha_1^2\alpha_2^2} = 4z^2\frac{(\alpha_2^2 - \alpha_1^2)}{\alpha_1^2\alpha_2^2}$$

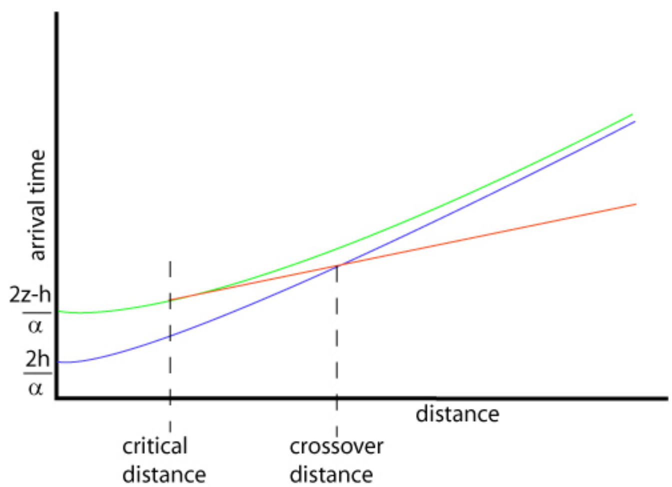
$$x^2(\alpha_2 - \alpha_1)^2 = 4z^2(\alpha_2 + \alpha_1)(\alpha_2 - \alpha_1)$$

$$x_C = 2z\sqrt{\frac{(\alpha_2 + \alpha_1)}{(\alpha_2 - \alpha_1)}}$$

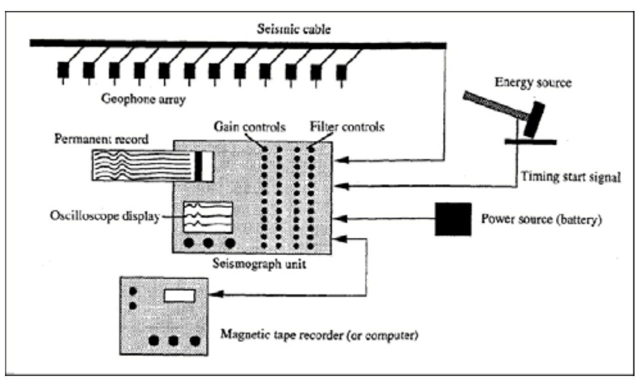
Typical Traveltime Curve

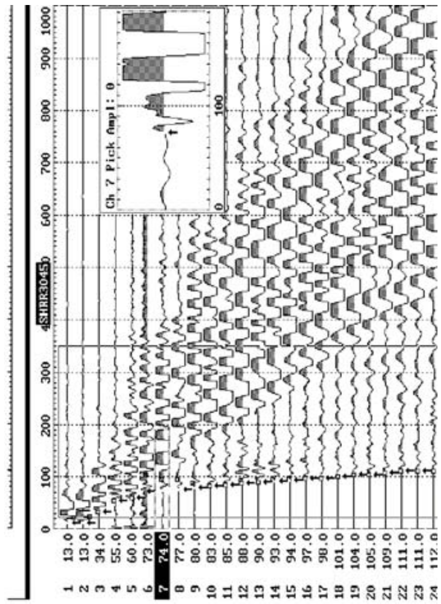


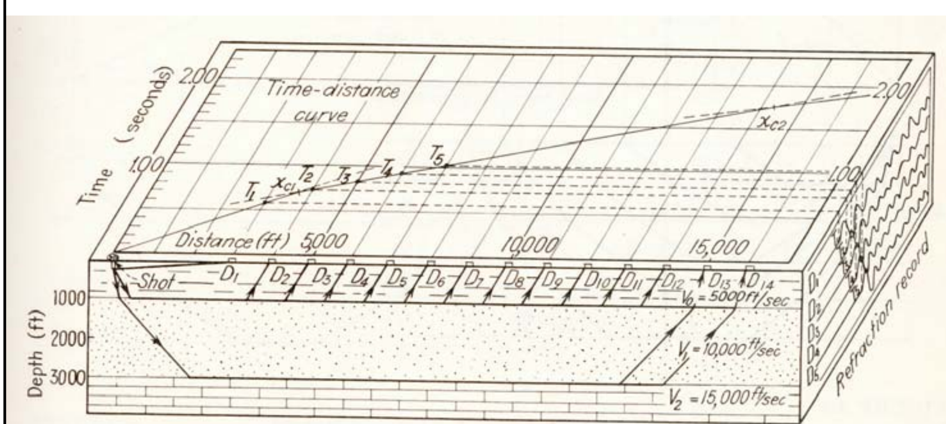
How does it change for a source not at the surface?

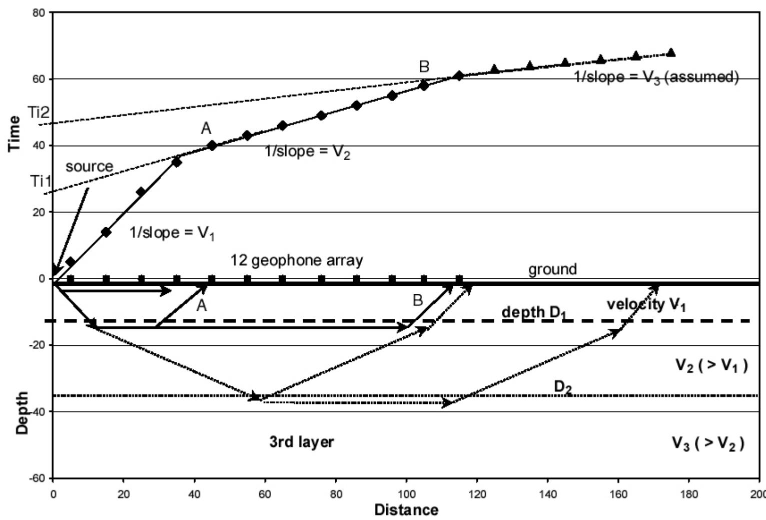


Seismic Refraction

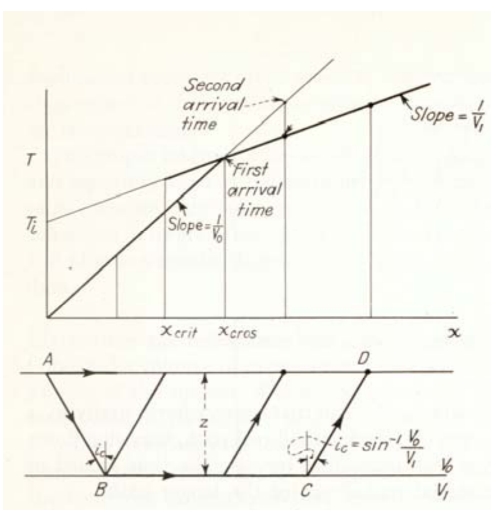





Time (seconds)
2.00
1.00
Time-distance curve
Distance (ft) 5,000
10,000
15,000
Shot
Depth (ft)
1000
2000
3000
$V_0 = 5000$ ft/sec
$V_1 = 10,000$ ft/sec
$V_2 = 15,000$ ft/sec
Refraction record



Two-Layer Refraction Problem

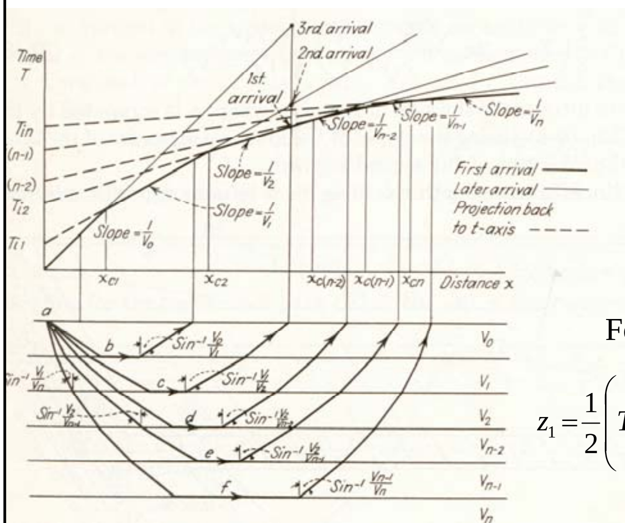


$$X_{crit} = 2z\, p/\eta_1 = 2z \frac{v_1}{\sqrt{v_2^2 - v_1^2}}$$

$$X_C = 2z\sqrt{\frac{v_2 + v_1}{v_2 - v_1}}$$

$$z = \frac{T_{i1} v_0 v_1}{2\sqrt{v_1^2 - v_0^2}}$$

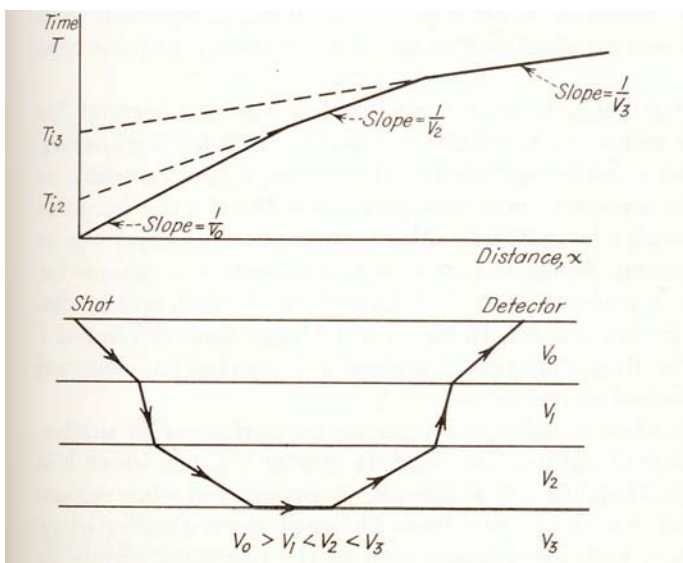
Multi-layer Problems



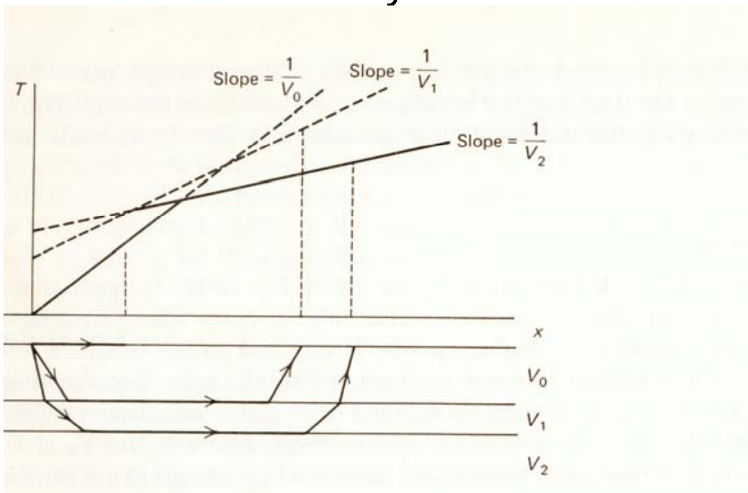
For three layers:

$$z_1 = \frac{1}{2}\left(T_{i2} - 2z_0 \frac{\sqrt{v_2^2 - v_0^2}}{v_2 v_0} \right) \frac{v_2 v_1}{\sqrt{v_2^2 - v_1^2}}$$

Low Velocity Layers Complications

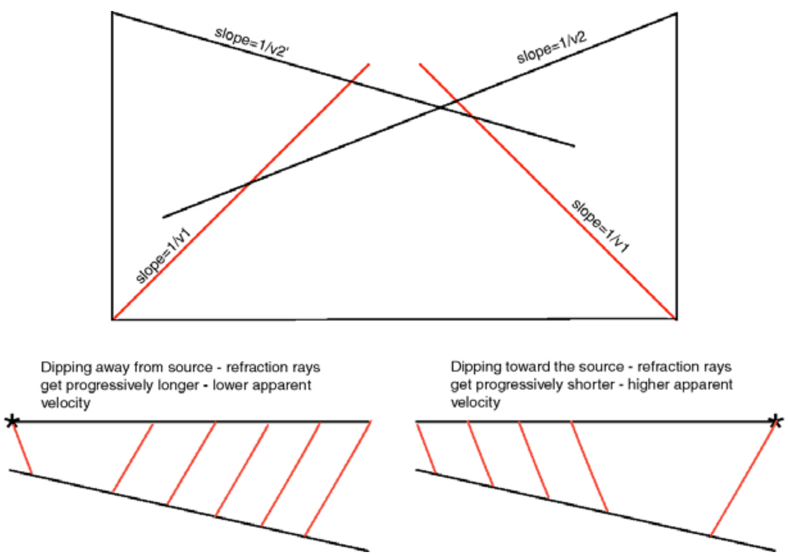


Hidden Layer Problem

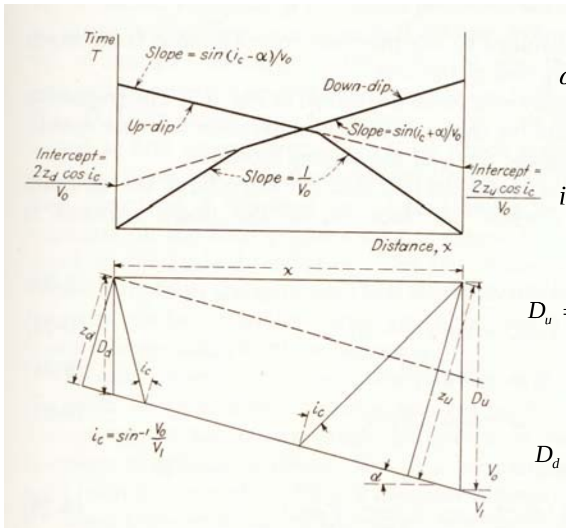


Both low velocity and hidden (thin) layers are complications that cannot be resolved from only first arrival time data. Secondary arrivals or amplitudes need to be employed.

Dipping Layer Problems



Dipping Layer Problems



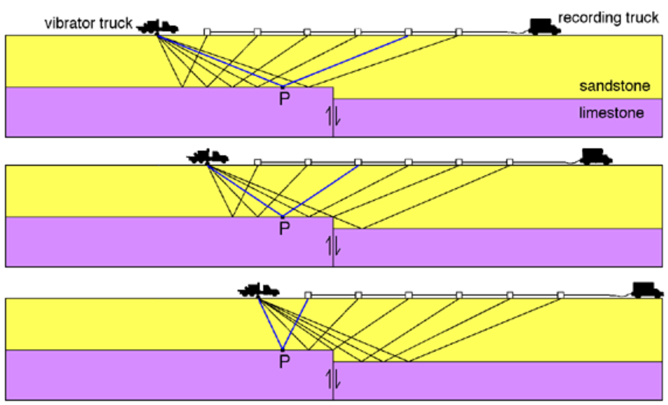
$$\alpha = \frac{1}{2}\left(\sin^{-1}\left(\frac{v_1}{v_d}\right) - \sin^{-1}\left(\frac{v_1}{v_u}\right)\right)$$

$$i_c = \frac{1}{2}\left(\sin^{-1}\left(\frac{v_1}{v_d}\right) + \sin^{-1}\left(\frac{v_1}{v_u}\right)\right)$$

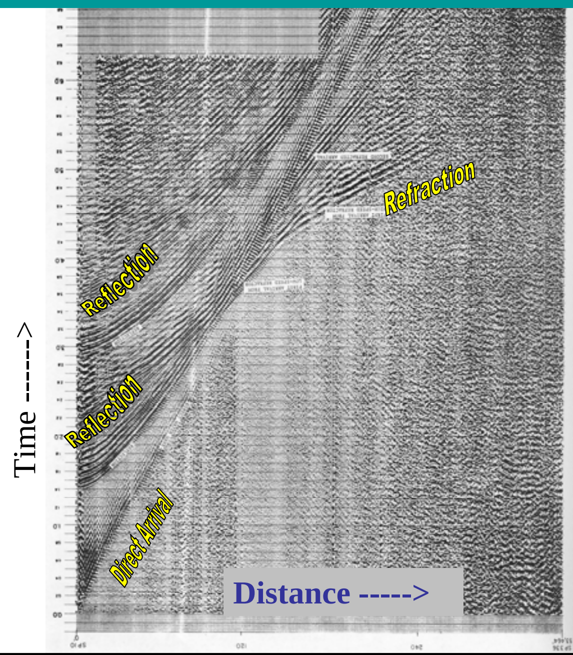
$$D_u = \frac{T_{iu} v_0 v_1}{2\cos(\alpha)\sqrt{v_1^2 - v_0^2}} = \frac{T_{iu} v_0}{2\cos(\alpha)\cos(i_c)}$$

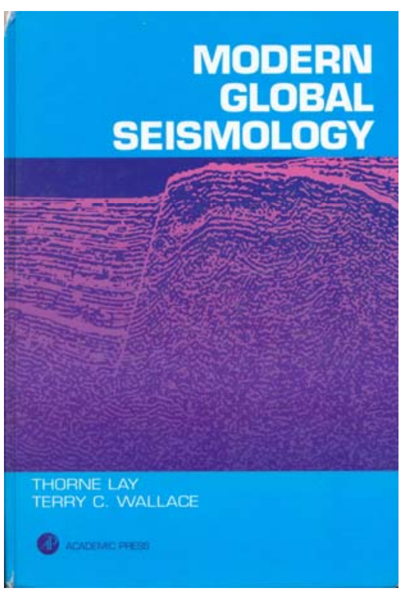
$$D_d = \frac{T_{id} v_0 v_1}{2\cos(\alpha)\sqrt{v_1^2 - v_0^2}} = \frac{T_{id} v_0}{2\cos(\alpha)\cos(i_c)}$$

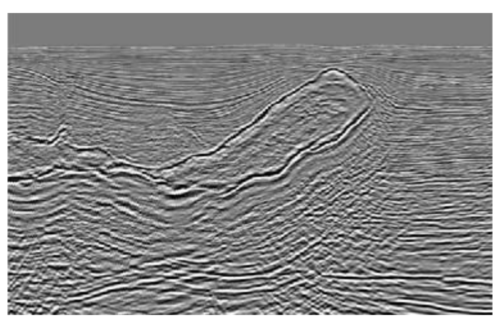
Seismic Reflection Method



Seismic Record Section

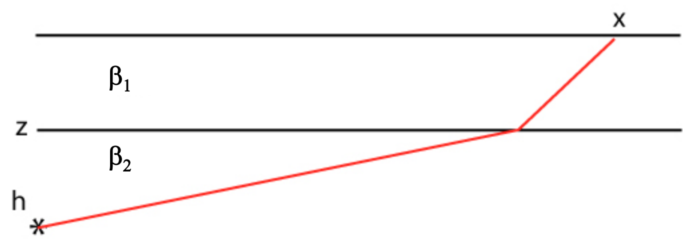




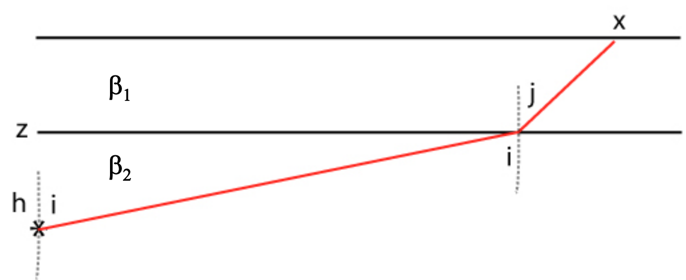


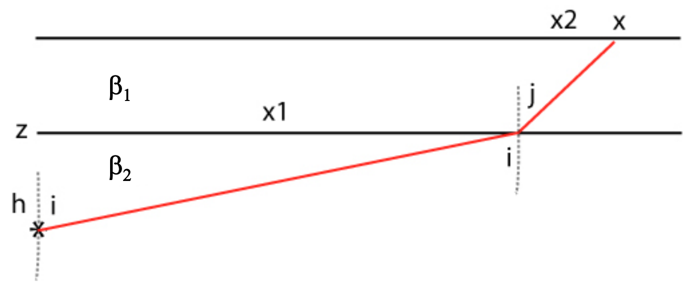
Reflection image courtesy of ChevronTexaco and WesternGeco.

Homework 4



How do we set up this problem to find arrival time and wave amplitude?





$$= (h - z) \cdot \tan(i) + z \cdot \tan(j)$$

$$= (h - z) \cdot \tan(i) + z \cdot \tan(\sin^{-1}(\frac{\beta_1}{\beta_2}\sin(i)))$$

$$x = (h - z) \cdot \tan(i) + z \cdot \tan(\sin^{-1}(\frac{\beta_2}{\beta_1}\sin(i)))$$

Find i

$$p = \frac{\sin(i)}{\alpha_2}$$

Calculate ray parameter

$$\tau = px + \eta_{\alpha_2}(h - z) + \eta_{\alpha_1} z$$

Calculate arrival time

$$T(p) = \frac{2\mu_2\eta_2}{\mu_1\eta_1 + \mu_2\eta_2}$$

Calculate transmission coefficient for amplitude

How about the setup for first multiple arrival?