

$$U_P(r, t) = \frac{1}{4\pi\rho r\alpha^3} R^P \dot{M} \left( t - \frac{r}{\alpha} \right)$$

$$U_{SV}(r, t) = \frac{1}{4\pi\rho r\beta^3} R^{SV} \dot{M} \left( t - \frac{r}{\beta} \right)$$

$$U_{SH}(r, t) = \frac{1}{4\pi\rho r\beta^3} R^{SH} \dot{M} \left( t - \frac{r}{\beta} \right)$$

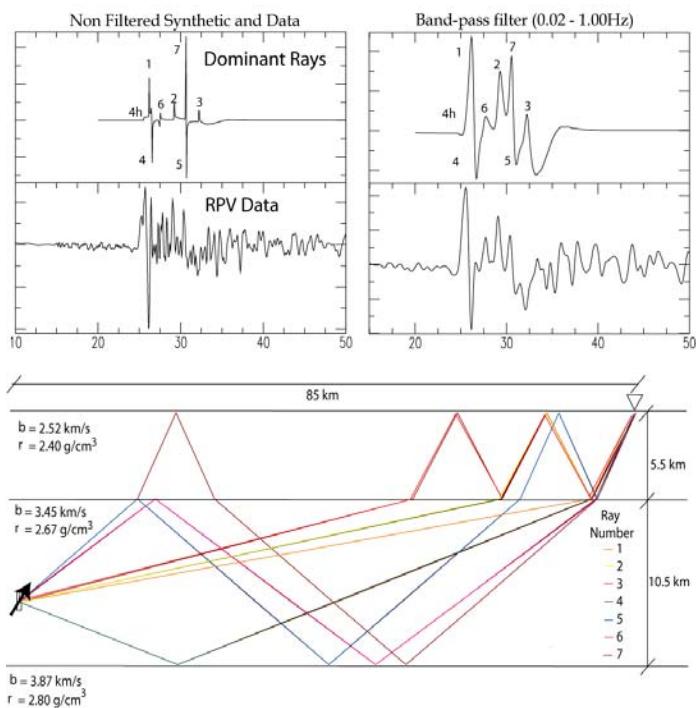
**What is assumed?**

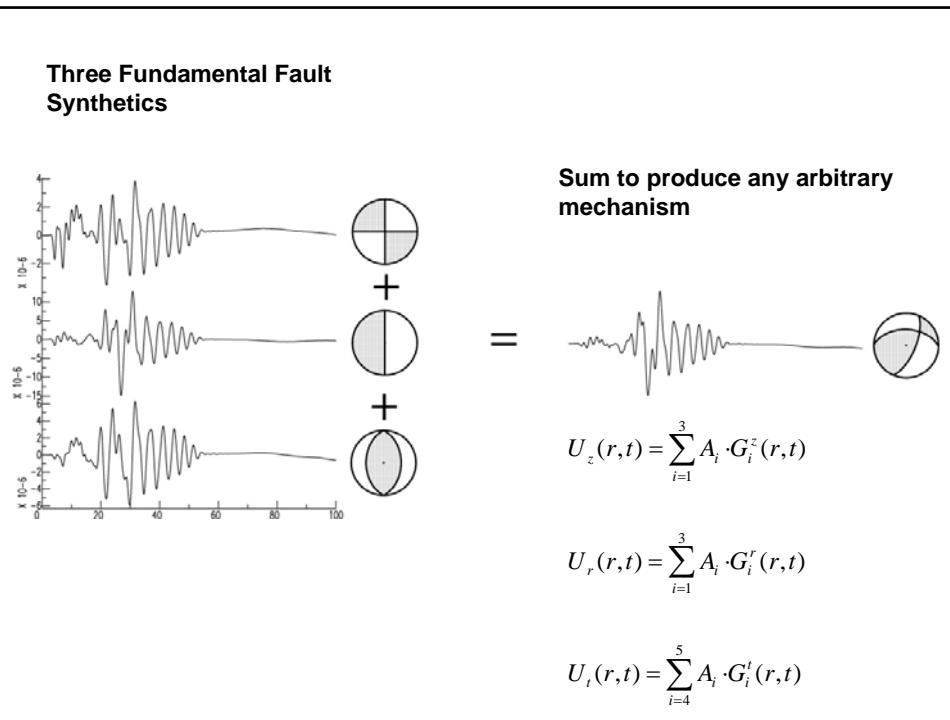
$$U_{SH}(r,t) = \frac{R_{SH} \dot{M}(t - r/\beta)}{4\pi\rho\beta^3 r} \quad \text{Whole Space}$$

$$U_{SH}(r,t) = \frac{2 \cdot R_{SH} \dot{M}(t - r/\beta)}{4\pi\rho\beta^3 r} \quad \text{Half Space}$$

$$\frac{2 \cdot \dot{M}(t - r/\beta)}{4\pi\rho\beta^3 r} R_{SH} = U_{SH}(r,t) \quad \text{Rewriting}$$

$$Ax = d \quad \text{Linear Equation}$$





$$U_z(r,t) = \sum_{i=1}^3 A_i \cdot G_i^z(r,t)$$

$$U_r(r,t) = \sum_{i=1}^3 A_i \cdot G_i^r(r,t)$$

$$U_t(r,t) = \sum_{i=4}^5 A_i \cdot G_i^t(r,t)$$

**There are 5 independent scaling coefficients (A)**

**The A coefficients are functions of station azimuth, strike, dip and rake.**

## Approximations of the Representation Theorem

$$u_n(t, \vec{x}) = \int d\tau \iint [u_i(\vec{\zeta}, \tau) \hat{v}_j C_{ijkl}] \cdot G_{nk,l}(\vec{x}, t - \tau; \vec{\zeta}, 0) d\Sigma$$

$$u_n(t, \vec{x}) = \int [u_i(\tau) \hat{v}_j C_{ijkl} \Sigma] \cdot G_{nk,l}(\vec{x}, t - \tau) d\tau \quad \text{Spatial point-source}$$

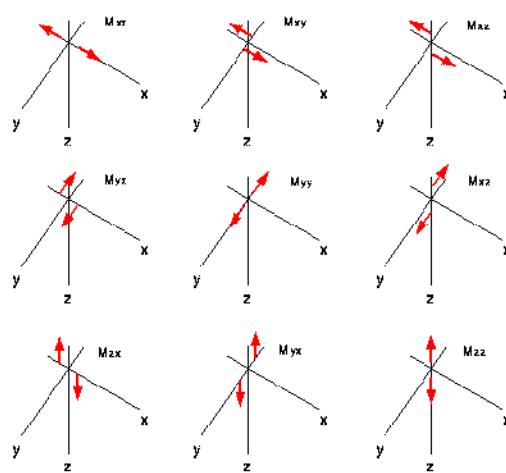
$$u_n(t, \vec{x}) = [u_i \hat{v}_j C_{ijkl} \Sigma] \cdot G_{nk,l}(\vec{x}, t) \quad \text{Spatial and temporal point-source}$$

$$u_n(t, \vec{x}) = M_{ij} \cdot G_{ni,j}(\vec{x}, t)$$

M has units of moment. i and j refer to directions of forces and derivatives. i.e. they define couples

$$U_n(\vec{x}, t) = M_{ij} \cdot G_{ni,j}(\vec{x}, t)$$

$$M_{ij} = \begin{pmatrix} m_{xx} & m_{xy} & m_{xz} \\ m_{yx} & m_{yy} & m_{yz} \\ m_{zx} & m_{zy} & m_{zz} \end{pmatrix}$$



What Kind of Mechanism is this?

$$M_{ij} = \begin{pmatrix} 0 & M_0 & 0 \\ M_0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

What are the principle axes?

Find Eigenvalues

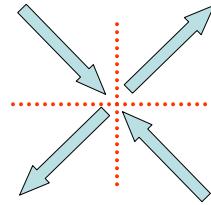
$$\det \begin{pmatrix} -\lambda & M_0 & 0 \\ M_0 & -\lambda & 0 \\ 0 & 0 & -\lambda \end{pmatrix} = 0$$

$$-\lambda(\lambda^2) - M_0(-\lambda M_0) = \lambda^2 - M_0^2 = 0$$

$$\lambda = \pm M_0$$

## Find Eigenvectors

$$\begin{pmatrix} -\lambda_i & \mathbf{M}_0 & 0 \\ \mathbf{M}_0 & -\lambda_i & 0 \\ 0 & 0 & -\lambda_i \end{pmatrix} \vec{a}_i = 0$$



$$\begin{pmatrix} -\mathbf{M}_0 & \mathbf{M}_0 & 0 \\ \mathbf{M}_0 & -\mathbf{M}_0 & 0 \\ 0 & 0 & -\mathbf{M}_0 \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = 0 \quad \hat{a} = \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0 \right\rangle$$

$$\begin{pmatrix} \mathbf{M}_0 & \mathbf{M}_0 & 0 \\ \mathbf{M}_0 & \mathbf{M}_0 & 0 \\ 0 & 0 & \mathbf{M}_0 \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = 0 \quad \hat{a} = \left\langle \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0 \right\rangle$$

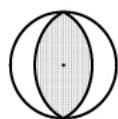
What Kind of Mechanism is this?

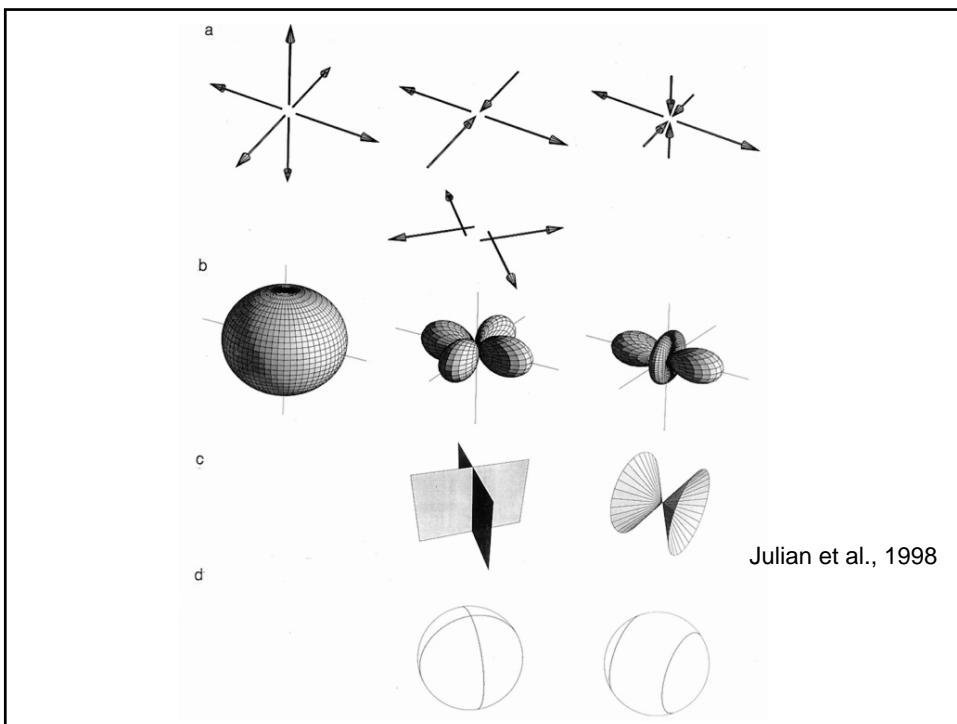
$$\mathbf{M}_{ij} = \begin{pmatrix} \mathbf{M}_0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\mathbf{M}_0 \end{pmatrix}$$

**What are the principle axes?**

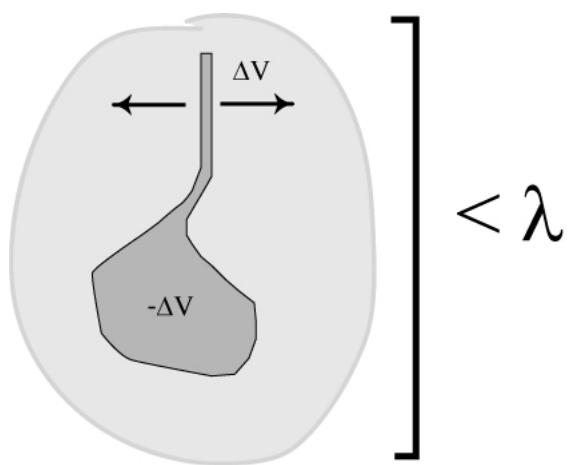
- Moment tensor is real and symmetric
- It can be diagonalizable to obtain eigenvalues and eigenvectors
- It can be decomposed into fundamental types (double-couple, compensated linear vector dipole, isotropic)
- Decomposition is non-unique (i.e 3DC, 3CLVD, DC+CLVD, Major DC + Minor DC, etc.....)

$$\begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} -2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

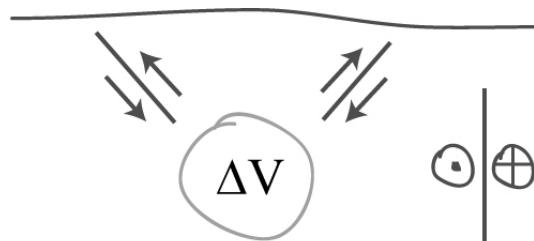




### Possible CLVD Mechanism?



## Nuke Mechanism



Collapse

|                 |  |  |   |  |                 |   |  |  |   |  |  |
|-----------------|--|--|---|--|-----------------|---|--|--|---|--|--|
| (a)             | $\begin{matrix} 6.28 \\ -0.67 \\ -1.29 \end{matrix}$ |  | = | $\begin{matrix} 1.44 \\ 1.44 \\ 1.44 \end{matrix}$   |                 | + | $\begin{matrix} 4.84 \\ -2.12 \\ -2.73 \end{matrix}$ |  |   |  |  |
| COMPLETE TENSOR |  |  |   | ISOTROPIC PART                                       | DEVIATORIC PART |   |  |  |   |  |  |
| (b)             | $\begin{matrix} 4.84 \\ -2.12 \\ -2.73 \end{matrix}$ |  | = | $\begin{matrix} 4.84 \\ 0 \\ -4.84 \end{matrix}$     |                 | + | $\begin{matrix} 0 \\ -2.12 \\ 2.12 \end{matrix}$     |  |   |  |  |
| DEVIATORIC PART |  |  |   | MAJOR DC   | MINOR DC        |   |  |  |   |  |  |
| (c)             | $\begin{matrix} 4.84 \\ -2.12 \\ -2.73 \end{matrix}$ |  | = | $\begin{matrix} 4.84 \\ 0 \\ -4.84 \end{matrix}$     |                 | + | $\begin{matrix} 0 \\ -2.73 \\ -2.73 \end{matrix}$    |  |   |  |  |
| DEVIATORIC PART |  |  |   | MAJOR DC   | MINOR DC        |   |  |  |   |  |  |
| (d)             | $\begin{matrix} 4.84 \\ -2.12 \\ -2.73 \end{matrix}$ |  | = | $\begin{matrix} 2.73 \\ 0 \\ -2.73 \end{matrix}$     |                 | + | $\begin{matrix} 2.12 \\ -2.12 \\ 0 \end{matrix}$     |  |   |  |  |
| DEVIATORIC PART |  |  |   | DCS WITH SAME T AXES                                 |                 |   |  |  |   |  |  |
| (e)             | $\begin{matrix} 4.84 \\ -2.12 \\ -2.73 \end{matrix}$ |  | = | $\begin{matrix} 3.79 \\ 0 \\ -3.79 \end{matrix}$     |                 | + | $\begin{matrix} 1.06 \\ -1.06 \\ 1.06 \end{matrix}$  |  |   |  |  |
| DEVIATORIC PART |  |  |   | DC   | CLVD            |   |  |  |   |  |  |
| (f)             | $\begin{matrix} 4.84 \\ -2.12 \\ -2.73 \end{matrix}$ |  | = | $\begin{matrix} 0.61 \\ 0 \\ -0.61 \end{matrix}$     |                 | + | $\begin{matrix} 4.23 \\ -2.12 \\ -2.12 \end{matrix}$ |  |   |  |  |
| DEVIATORIC PART |  |  |   | DC   | CLVD            |   |  |  |   |  |  |
| (g)             | $\begin{matrix} 4.84 \\ -2.12 \\ -2.73 \end{matrix}$ |  | = | $\begin{matrix} 0 \\ -0.31 \\ -0.31 \end{matrix}$    |                 | + | $\begin{matrix} 4.84 \\ -2.42 \\ -2.42 \end{matrix}$ |  |   |  |  |
| DEVIATORIC PART |  |  |   | DC   | MAXIMUM CLVD    |   |  |  |   |  |  |
| (h)             | $\begin{matrix} 4.84 \\ -2.12 \\ -2.73 \end{matrix}$ |  | = | $\begin{matrix} 2.52 \\ -2.52 \\ -2.52 \end{matrix}$ |                 | + | $\begin{matrix} 2.32 \\ -2.32 \\ 0 \end{matrix}$     |  | + | $\begin{matrix} 0 \\ 0.20 \\ -0.20 \end{matrix}$ |  |
| DEVIATORIC PART |  |  |   | THREE DCS  |                 |   |  |  |   |  |  |

Julian et al., 1998

