Problem 1

Poisson's model: $P(x \geq 1) = 1.0 - e^{-\lambda t}$

Gutenberg-Richter: $Log(N) = 3.17 - 0.793 \cdot M$

M = 5.0	λ = N = 1.60E-01	→	6.24	year event
M = 7.0	λ = N = 4.16E-03	→	240	year event
M = 8.0	λ = N = 6.70E-04	→	1493	year event

Poisson

Magnitude = 5			
t (year) =	1/52	p (x>1) =	0.31%
t (year) =	1/12	p (x>1) =	1.33%
t (year) =	1	p (x>1) =	14.81%
t (year) =	5	p (x>1) =	55.14%

Magnitude = 7			
t (year) =	1/52	p (x>1) =	0.01%
t (year) =	1/12	p (x>1) =	0.03%
t (year) =	1	p (x>1) =	0.42%
t (year) =	5	p (x>1) =	2.06%

Magnitude = 8			
t (year) =	1/52	p (x>1) =	0.00%
t (year) =	1/12	p (x>1) =	0.01%
t (year) =	1	p (x>1) =	0.07%
t (year) =	5	p (x>1) =	0.33%

As expected, the probabilities of occurrence increase with increasing time interval and decrease with magnitude.

If we compare the results of the M8 event calculated with the Gutenberg-Richter model, the magnitude 8 event will represent nearly the 1500 year event.

On the other hand, if the characteristic earthquake is used with an average slip of 450 cm and a loading rate of 1.9 cm/year, the recurrence interval of the M8 event will drastically change. In fact:

$$\frac{slip}{rate} = \frac{450 \ \ cm}{1.9 \ \ {}^{cm}\!/_{year}} = 237 \ \ year\ event$$

More than a factor of 6 difference between the two models.

This example shows the limitation of extrapolating the Gutenberg-Richter model to large magnitudes. Also, in this model the fact that faults tend to release energy in characteristic earthquakes is not taken into account.

When forecasting earthquakes, it is of extreme importance to assume an appropriate recurrence interval. If the Gutenberg-Richter model was used to forecast earthquakes in the north coast SAF M8 event, an event every 1500 years would be expected, instead of every 240 years, as the characteristic earthquake model shows.

Problem 2

The aggregate probability is written as: $P[A \cup B \cup C ... \cup N]$

Where for two events is: $P[A \cup B] = P[A] + P[B] - P[AB]$

If the events are independent: $P[A \cup B] = P[A] + P[B] - P[A] \cdot P[B]$

If each event can be represented as a Poisson Process:

$$P[A \cup B] = \left(1 - e^{-rate(A)}\right) + \left(1 - e^{-rate(B)}\right) - \left(1 - e^{-rate(A)}\right)\left(1 - e^{-rate(B)}\right) = 1 - e^{-rate(A) - rate(b)}$$

And for N processes: $P[A \cup B \cup C ... \cup N] = 1 - e^{-\Sigma x_i}$

In our case, as Δt and ΔM tend to zero and for the given intervals M1, M2 and T1, T2:

$$P[M1, M2, T1, T2] = 1 - e^{-\int_{M1}^{M2} \int_{T1}^{T2} rate(m,t) \cdot dm \cdot dt}$$

Reasenberg and Jones: $rate(t,M) = 10^{-1.67+.091\cdot(M_m - M)} \cdot (t+0.05)^{-1.08}$

Defining $\Delta\lambda \cdot \Delta t = \int_{M1}^{M2}\int_{t1}^{t2} rate(t,M) \cdot dt \cdot dM$

The probability of an M5 or larger in the 7 days following the main event (M5.8) can be calculated:

$$\Delta\lambda \cdot \Delta t = \int_{5}^{\infty}\int_{0.1}^{7} 10^{-1.67+.091\cdot(5.8-M)} \cdot (t+0.05)^{-1.08} \cdot dt \cdot dM$$

The limits for the integration for the magnitude cannot go to infinity due to fault physical limitations, but after an M8, the values of this function are so low that we can assume they are equal to zero.

The double integral above can be separated into 2 integrals:

$$\Delta\lambda \cdot \Delta t = \int_{5}^{8} 10^{-1.67+.091\cdot(5.8-M)} \cdot dM \cdot \int_{0.1}^{7} (t+0.05)^{-1.08} \cdot dt = 0.0545 \text{ x } 3.857 = 0.21$$

Therefore the probability can be computed as: $P[5,8,0.1,7] = 1.0 - e^{-0.21} = 18.9\%$

After 2 days, the probability of occurrence of a magnitude +5 aftershock decays to:

$$\Delta\lambda \cdot \Delta t = \int_{5}^{8} 10^{-1.67+.091\cdot(5.8-M)} \cdot dM \cdot \int_{2}^{9} (t+0.05)^{-1.08} \cdot dt = 0.0545 \text{ x } 1.110 = 0.0705$$

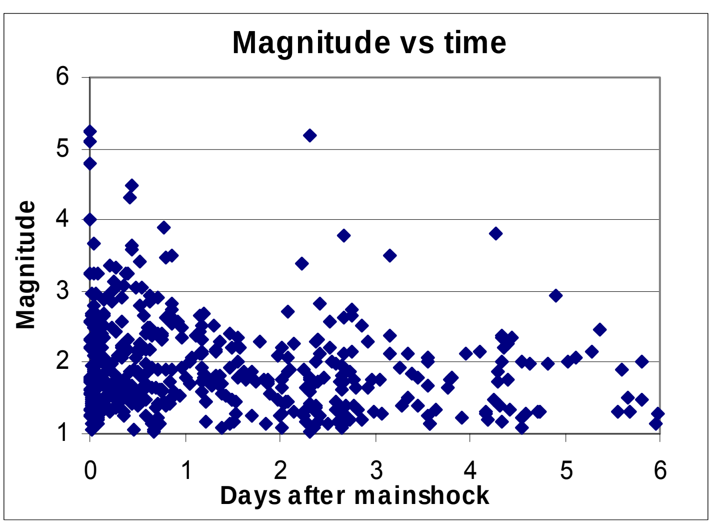
$$P[5,8,2,9] = 1.0 - e^{-0.0705} = 6.80\%$$

This is a decrease in the probability of nearly 12%

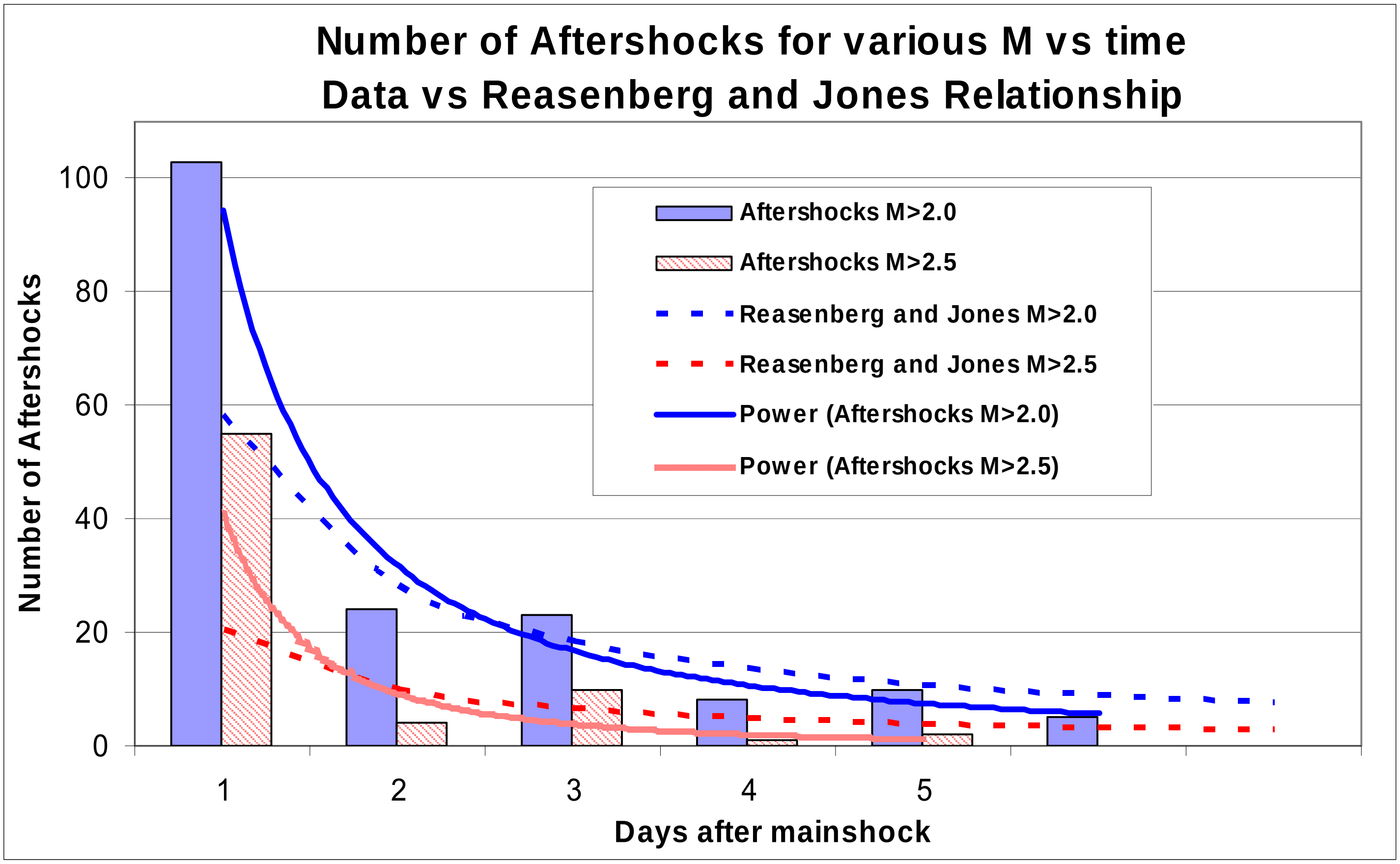
The statistics compiled by Reasenberg and Jones allow us to estimate the probability of an event greater than the mainshock in the following 7 days:

$$\Delta\lambda \cdot \Delta t = \int_{5.8}^{\infty} 10^{-1.67+.091\cdot(5.8-M)} \cdot dM \cdot \int_{0.1}^{7} (t+0.05)^{-1.08} \cdot dt = 0.0102 \text{ x } 3.857 = 0.039$$

$$P[5.8,8,0.1,7] = 1.0 - e^{-0.039} = 3.86\%$$



The following plots show the distribution of the recorded aftershocks. The one on the left shows several strong aftershocks (two of them above M5, one of which happened just a minute after the mainshock and the second one, more than two days after the initial event). The bottom plot shows the trend of the measured aftershocks of M>2.0 and M>2.5, compared to the rate of aftershocks predicted by the Reasenberg and Jones decay rate for a generic California earthquake. The trend lines show how the Reasenberg and Jones relationship does a very good job in predicting of the number of aftershocks.



Several of you argued that since there were 450 aftershocks of M ≥ 1 "recorded" during the first 7 days following the mainshock, and only 2 of these events had M ≥ 5, then the previously calculated probabilities were off. This is comparing two different things. A 19% probability of an M ≥ 5 means that according to the "generic" prediction, if 100 events with M5.8 happen, then in 19 of those events you will see an aftershock of magnitude greater or equal to M5. This is completely different from saying that if you record all the aftershocks that happen after the event, then 19% of them will be greater than M5. Also, the aftershocks that were recorded were those of M ≥ 1, this does not mean that those were all the aftershocks. There may have been many that were not recorded by the sensors.

AFTERSHOCK FORECAST

January 24, 1980

This forecast is based on the statistics of aftershocks typical for California. This is not an exact prediction, but only a rough guide to expected aftershock activity.

MAINSHOCK: January 24, 1980 MAGNITUDE 5.8

STRONG AFTERSHOCKS (Magnitude 5 and larger)

At this time (2 hours and a half after the mainshock) the probability of a strong and possibly damaging aftershock IN THE NEXT 7 DAYS is approximately **15 to 20%**

EARTHQUAKES LARGER THAN THE MAINSHOCK

Most likely, the recent mainshock will be the largest in the sequence. However, there is a small chance (APPROXIMATELY **2** to **5%**) of an earthquake equal to or larger than this mainshock in the next 7 days.

Problem 3

The data on the right summarizes the years of great M8 earthquakes on the Mojave segment of the SAF.

This data was compared with two possible distributions: Normal and LogNormal.

Using the Mean and Variance from above, and the equations given in the problem set, we can calculate the probability distributions (Normal and LogNormal) that have these parameters.

DATA

Date	Δt
1857	45
1812	332
1480	134
1346	246
1100	52
1048	51
997	200
797	63
734	63
671	142
529	
Mean =	132.8
σ =	98.8

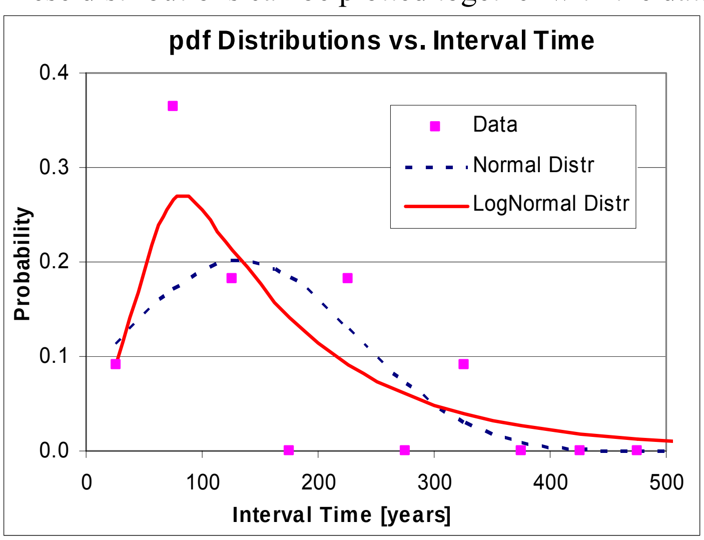
HISTOGRAM

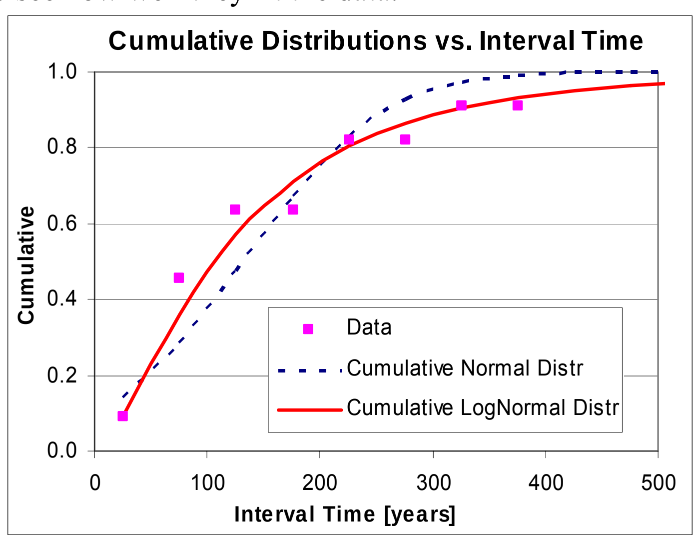
Δt	n	n/(1+N)	Cumulative
26	1	0.1	0.091
76	4	0.4	0.455
126	2	0.2	0.636
176	0	0.0	0.636
226	2	0.2	0.818
276	0	0.0	0.818
326	1	0.1	0.909
376	0	0.0	0.909
426	0	0.0	0.909
476	0	0.0	0.909
526	0		

Note: The pdf equation that is given in the assignment is for *continuous* functions, while we are dealing with sampled data (*discontinuous*). We need to take this fact into account by multiplying the probability value given by the equation times the sampling interval (50 years). In fact, this will give an area of 1.0 for the distributions.

Δt	pdf		Normalized pdf		Cumulative	
	Normal	LogNormal	Normal	LogNormal	Normal	LogNormal
26	0.0023	0.0019	0.113	0.093	0.14	0.093
76	0.0034	0.0053	0.171	0.266	0.28	0.359
126	0.0040	0.0042	0.202	0.212	0.47	0.572
176	0.0037	0.0028	0.184	0.142	0.67	0.714
226	0.0026	0.0018	0.129	0.092	0.83	0.806
276	0.0014	0.0012	0.071	0.060	0.93	0.866
326	0.0006	0.0008	0.030	0.040	0.97	0.905
376	0.0002	0.0005	0.010	0.027	0.99	0.932
426	0.0000	0.0004	0.002	0.018	1.00	0.950
476	0.0000	0.0003	0.000	0.013	1.00	0.963
526	0.0000	0.0002	0.000	0.009	1.00	0.973

These distributions can be plotted together with the data to see how well they fit the data.





These plots show a better agreement of the LogNormal distribution with respect to the data. The left plot shows well the better match in the mode of the data and the distribution (at around 90 years interval), while the right plot clearly shows a better fit of the LogNormal distribution as a whole.

Problem 4

To determine the 30-year probability of magnitude +8 in 1999 we can apply the following relationships, based on the distributions previously estimated:

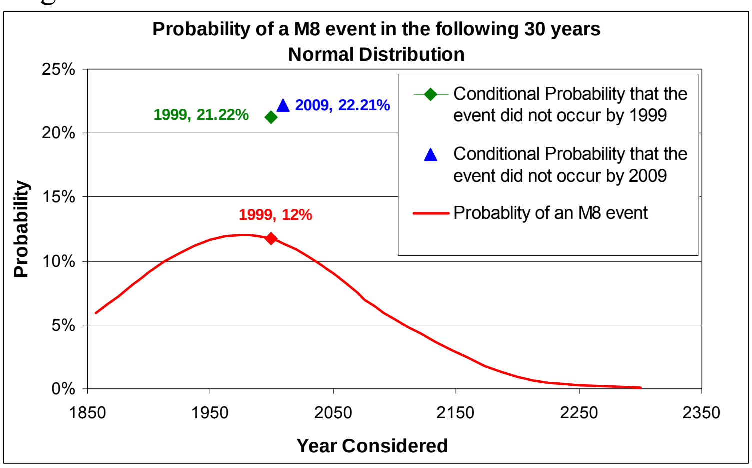
$$P[Te \le T \le Te + \Delta T] = \int_{Te}^{Te+\Delta T} pd(u) \cdot du$$

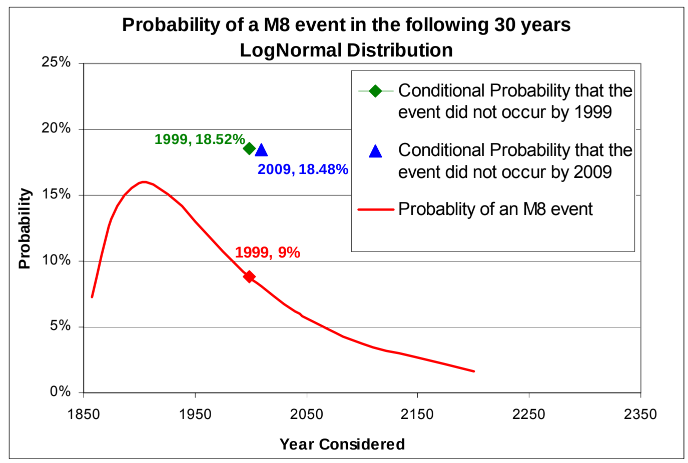
where *Te* is the time since the previous event, *ΔT* = 30 years.

This probability does not consider the condition that the event did not occur before Te. To take this into account, the probability of the event in a given time window can be estimated from:

$$P[Te \le T \le Te + \Delta T | T \ge Te] = \frac{\int_{Te}^{Te+\Delta T} pd(u) \cdot du}{1.0 - \int_{0}^{Te} pd(u) \cdot du}$$

Applying both this probabilities formulations to several years, we can estimate the likelihood of having the magnitude +8 event in 30-year time windows. This is shown in the figures bellow for the Normal and LogNormal distributions.





A mayor difference that can be observed from the two plots, is that assuming a LogNormal distribution, it appears clear that the event has not occur for a time longer than most recurrent time, while assuming a Normal distribution, the event is at its largest probability of occurring in the next 30years.

The figures also show the conditional probabilities of the earthquake happening in a 30-year window, given that it has not occur by 1999 and then given that it has not occur by 2009. It is shown that the probability will increase, but very slight.

The probability of occurrence can be also calculated with the Poisson model. This model has "no memory" of previous events. The following table shows the results of the prediction using the time dependent models and the Poisson model for 3 time windows in 1999:

	Normal Conditional		Poisson Model		LogNormal Conditional	
	Te = 142		λ = 132.8 year event		Te = 142	
Date Evaluated	Δt	Probability	Δt	Probability	Δt	Probability
1999	10	7.24%	10	7.25%	10	7.30%
1999	20	14.33%	20	13.98%	20	13.44%
1999	30	21.22%	30	20.22%	30	18.52%

This clearly shows that the two methods agree very closely for the studied period. This agreement is in general very good in zones close to the average return period for a fault (like it is in this case, where the average return period is nearly 130 years and the elapsed time since the last rupture is nearly 140 years).