Discrimination of the flow law for subglacial sediment using in situ measurements and an interpretation model

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Abstract. Subglacial hydrological and mechanical processes play a critical role in determining the flow characteristics and stability of glaciers and ice sheets. An improved understanding of these mechanisms is necessary if we are to accurately predict the response of ice masses to changes in climatic forcings. For this purpose, we have taken simultaneous measurements of basal water pressure, pore-water pressure, sediment deformation, glacier sliding, and sediment strength beneath Trapridge Glacier, Yukon Territory, Canada. To interpret these data we have developed a simple hydromechanical model of the processes acting beneath a soft-bedded alpine glacier. In this model, the glacier bed is categorized into three distinct regions: soft-bedded regions that are hydraulically connected to the subglacial drainage system; soft-bedded but hydraulically-unconnected regions; and hard-bedded regions. Each basal region is represented in the model as a one-dimensional column. The time evolution of pore-water pressure, till dilatancy, sediment deformation and glacier sliding is calculated in the two soft-bedded columns, while the hard-bedded region is considered rigid and impermeable. The three columns are coupled by a simple ice-dynamics model that accounts for water-pressure-driven changes in basal shear stress distribution. Synthetic responses for the instruments used beneath Trapridge Glacier are calculated from the modeled basal conditions, providing a framework for improving interpretation of the field records.

The model is used to determine which of several till flow laws proposed in the literature best represents conditions beneath Trapridge Glacier. Investigated are linear-viscous, nonlinear-viscous, nonlinear-Bingham and Coulomb-plastic tills. Model parameter values are chosen to simulate typical summer conditions beneath Trapridge Glacier, and pore-water pressures, sediment deformation profiles, sliding rates and basal shear stress values are calculated for each flow law. Comparison of synthetic and field instrument responses for each flow law suggests that till behavior is best represented as Coulomb-plastic.

1. Introduction

Sediment deformation and basal sliding can contribute greatly to the motion of ice masses that are underlain by water-saturated sediments. Basal motion is the dominant flow mechanism of some alpine glaciers [e.g., *Raymond*, 1971; *Engelhardt et al.*, 1978; *Boulton and Hindmarsh*, 1987] and provides the key to understanding fast flow of ice streams [e.g., *Blankenship et al.*, 1986; *Alley et al.*, 1987a,b], tidewater glaciers [*Meier and Post*, 1987] and surging glaciers [e.g., *Meier and Post*, 1969; *Kamb et al.*, 1985; *Fowler*, 1987; *Kamb*, 1987; *Raymond*, 1987]. Much recent emphasis has been placed on the flow dynamics of Antarctic ice streams, which play an important role in the mass balance of the potentially-unstable West Antarctic Ice Sheet [*Mercer*, 1978] and provide modern analogues to fast-flowing regions of the Laurentide and Cordilleran Ice Sheets inferred from geologic evidence [e.g., *Morner and Dreimanis*, 1973; *Clayton and Moran*, 1982; *Brown et al.*, 1987]. Despite the fundamental role basal motion plays in glacier dynamics, our understanding of the underlying processes remains incomplete.

Studies of ice-mass dynamics have used a wide range of flow laws for describing the behavior of subglacial sediments, from linear viscous [e.g., *Alley et al.*, 1987b; *MacAyeal*, 1989] to rate-independent [e.g., *Tulaczyk et al.*, 2000a]. Subglacial sediments with different rheological behaviors would respond differently to a given set of subglacial forcings. In situ measurements made by subglacial instruments provide a means of observing these responses. During the summer of 1996, instruments installed at the bed of Trapridge Glacier, Yukon Territory, Canada (Fig. 1) recorded simultaneous measurements of basal water pressure, pore-water pressure, sediment strain rate, basal sliding and sediment strength. These records present a picture of subglacial conditions complete enough to allow us to investigate whether they could aid in determining which of the proposed flow laws best describes the mechanical behavior of subglacial sediments.

To this end, we have developed a simple hydromechanical model of the processes governing basal motion of a soft-bedded alpine glacier. In the model, the glacier bed is classified into three general regions, representing (1) regions that are underlain by deformable sediments and hydraulically connected to the subglacial drainage system, (2) regions with soft sediments that are poorly connected, and (3) hard-bedded regions. Each basal region is modeled as a

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single one-dimensional column. In the soft-bedded regions, the time-evolution of pore-water pressure, sediment dilatancy, sediment deformation and glacier sliding is calculated; the hard-bedded region is considered rigid and impermeable. Sediment properties are determined by the pore-water pressure, porosity and preconsolidation history of the sediment. The columns are coupled by a simple ice-dynamics model, allowing investigation of the effects of stress bridging between regions.

Sediment deformation profiles are calculated using four flow laws that have been proposed in the literature: linear-viscous, nonlinear-viscous, nonlinear-Bingham and Coulomb-plastic. Synthetic instrument responses are calculated from the modeled pore-water pressure, sediment deformation, sliding and sediment strength conditions. The magnitude and form of the calculated instrument responses are compared to the field records, allowing us to determine which sediment flow law provides the best qualitative match to the field measurements.

2. Field instrument records

Figure 2 shows simultaneous measurements of basal water pressure, pore-water pressure, sediment strain rate, basal sliding and sediment strength recorded by five instruments during the period 20–25 July 1996 at Trapridge Glacier, Yukon Territory, Canada (Fig. 1). The responses in Figure 2 exhibit typical magnitudes and phase relations (with respect to diurnal water pressure fluctuations) for each instrument type during the summer season at Trapridge Glacier. The five instruments were installed in three boreholes located within ~7.0 m of each other at the glacier bed. Sensors P1 and PL1 shared one hole; PZ1 and BT1 shared another (Fig 1c). Water level fluctuations in each borehole immediately following drilling indicates that all three boreholes were placed in hydraulically-connected regions of the bed, and the close proximity of the boreholes suggests that basal conditions and forcings were uniform over the region in which they were installed.

Figure 2a shows records for pressure transducers P1 and PZ1, with pressure expressed in units of pressure head $p_h = p/\rho_w g$. Pressure transducer P1 was suspended within the borehole and measured the water pressure in the subglacial drainage system; PZ1 was installed at a depth of ~0.15 m within the subglacial sediments. Indicated pressures are taken to represent the pore-water pressure within the sediments at this depth. The records for both P1 and PZ1 show strong diurnal pressure fluctuations, indicating continued hydraulic connection with the subglacial drainage system. The record for transducer P1 exhibits peak pressures close to the estimated flotation value of ~62 m. The record for PZ1 shows pressure variations that are slightly lower in amplitude than those indicated in the record for P1, and with a diurnal pressure cycle that is slightly lagged in phase in comparison with P1.

Figure 2b shows the strain rate in the subglacial sediments inferred from the tilt record for tilt cell BT1. The tilt cell, which measures 0.11 m long and incorporates pressure transducer PZ1 (Fig. 2a), was installed ~0.15 into the sediments. During the five-day period shown in Figure 2, the tilt sensor rotated approximately 14.7° (assumed down-glacier). The tilt angle time-series data were smoothed with a 3-hour moving-boxcar filter prior to differentiation. Strain rate variations are out-of-phase with water pressure fluctuations.

Figure 2c shows inferred glacier sliding rates from the record for slidometer SL1. The anchor was inserted ~0.12 m into the subglacial sediments. Prior to differentiation, raw sliding displacement data were smoothed with a 3-hour moving boxcar filter. The slightly negative sliding rates indicated in the record are clearly non-physical and are likely artifacts of the instrument design. Strong correlation between sliding rate and subglacial water pressure are observed during days 202–204; sliding rates during days 205–206 are not as clearly related to pressure variations.

Ploughmeter PL1 (Fig. 2d) was installed in the same borehole as pressure transducer P1 (Fig. 2a). The tip of PL1 was inserted approximately 0.14 m into the sediments. Measured forces exhibit diurnal variations that are out-of-phase with water pressure fluctuations.

3. Flow laws for subglacial sediments

The rheological nature of subglacial sediments remains as one of the more pressing questions in glaciology. Studies to date have variously concluded that the behavior of subglacial sediments is only slightly nonlinear [e.g., *Boulton and Hindmarsh*, 1987], highly nonlinear [e.g., *Kamb*, 1991; *Hooke et al.*, 1997] or rate-independent (typically described as "Coulomb-plastic"; [e.g., *Iverson et al.*, 1997; *Iverson et al.*, 1998; *Tulaczyk et al.*, 2000a]. Models of ice stream dynamics have generally assumed subglacial sediments to behave as a linear-viscous [*Alley et al.*, 1987b; *MacAyeal*, 1989] or mildly nonlinear *Alley*, 1989] fluid, although recent models have also considered Coulomb-plastic tills [e.g., *Tulaczyk et al.*, 2000b]. (Here and for the remainder of this manuscript, the word "till" is used to describe any unconsolidated subglacial sediments, not just those that are basally derived.). Because an ice mass underlain by a highly-nonlinear till is more prone to unstable behavior than is one resting on a linear-viscous till [*Alley*, 1990; *Kamb*, 1991], the flow characteristics of till play a critical role in the stability of ice sheets and glaciers.

In this study, we will investigate four commonly-used flow laws for subglacial sediments. The first assumes sediments to deform as a Newtonian viscous material (Fig. 4a):

$$\dot{\varepsilon} = \frac{1}{2\eta_0}\tau. \tag{1}$$

Here the strain rate $\dot{\varepsilon}$ is linearly related to the shear stress τ and the till viscosity η_0 is independent of pore-water pressure. This relation has been used to model the flow of ice streams [e.g., *Alley et al.*, 1987b; *Alley et al.*, 1989; *MacAyeal*, 1989] and alpine glaciers [e.g., *Fischer and Clarke*, 1994].

The second and third flow laws investigated were proposed by *Boulton and Hindmarsh* [1987]. These relations were derived from observations taken in near-margin sediments beneath Breidamerkurjökull, Iceland. The first of these relations considers sediments a nonlinear-viscous material (Fig. 4b):

$$\dot{\varepsilon} = B_1\, \tau^a\, (p')^{-b}. \tag{2}$$

As porewater pressures increase the effective pressure ($p' = p_I - p$, where p_I is the ice-overburden pressure and p the pore-water pressure) decreases, weakening the till. The second relation treats till as a Bingham material (Fig. 4c), in which the strain rate depends on the amount by which the shear stress exceeds the yield strength σ_Y:

$$\dot{\varepsilon} = B_2 (\tau - \sigma_Y)^a \, (p')^{-b}. \tag{3}$$

The yield strength σ_Y is determined by the Mohr-Coulomb failure criterion,

$$\sigma_Y = c_0 + p' \tan\phi, \tag{4}$$

in which c_0 is the cohesion and ϕ the friction angle. At stress values below σ_Y, the till behaves elastically; at values above the yield strength the till deforms viscously. The terms B_1, B_2, a, and b in (3) and (2) are tuning parameters.

The fourth flow law investigated assumes that till behaves as a Coulomb-plastic material (Fig. 4d). The strength of such a material (assuming that it has been sufficiently sheared to reach a residual state; see *Iverson et al.* [1998]) is linearly related to the effective pressure but independent of the strain rate. Laboratory studies by *Iverson et al.* [1998] and *Tulaczyk et al.* [2000a] suggest that till behavior can be approximated as Coulomb-plastic, and this behavior has been used to model basal motion of the Puget Lobe of the Cordilleran Ice Sheet [*Brown et al.*, 1987] and of both alpine glaciers [e.g., *Iverson*, 1999] and ice streams [e.g., *Tulaczyk et al.*, 2000b]. Highly nonlinear flow is also suggested by analysis of till from the base of Antarctic Ice Stream B by *Kamb* [1991], which yielded values of a as large as ~100; such highly nonlinear flow would be difficult to distinguish from purely plastic behavior.

At stresses below the yield strength σ_Y determined by the Mohr-Coulomb failure criterion (4), the behavior is elastic and no permanent deformation occurs, while flow at stresses greater than the yield strength is instantaneous:

$$\dot{\varepsilon} = \begin{cases} > 0 & \text{if } \tau \geq \sigma_Y \\ 0 & \text{if } \tau < \sigma_Y. \end{cases} \tag{5}$$

No direct relationship between shear stress and deformation rate exists for values of $\tau \geq \sigma_Y$: deformation occurs at the rate necessary to prevent the applied shear stress from exceeding σ_Y. The abrupt transition between non-deforming and deforming states for Coulomb-plastic till creates a high degree of numerical stiffness. To reduce this stiffness, we model Coulomb-plastic behavior as

$$\dot{\varepsilon} = \frac{\dot{\varepsilon}_0}{2} \left[1 + \tanh\left(2\pi \frac{\tau - \sigma_Y}{\Delta\tau} \right) \right]. \tag{6}$$

Assigning a sufficiently large value to the reference strain rate $\dot{\varepsilon}_0$ and a small value to the failure range $\Delta\tau$ allows close approximation to the Coulomb-plastic behavior of (5). (The hyperbolic tangent $\tanh(x)$ is an exponential, rather than circular, function. As such, the factor 2π has the effect of largely confining the transition between non-deforming and deforming states to the interval $\sigma_Y \pm \Delta\tau/2$. Without this factor, which plays a similar role in (13), the transition would occur over a wider range of shear stress values.)

4. The model

4.1. Basal representation and model geometry

Spatial variations in drainage system morphology and sediment properties lead to considerable nonuniformity in mechanical and hydraulic characteristics of the bed. While much complexity could be introduced to account for these variations and may eventually prove necessary to explain the broad range of subglacial phenomena observed in instrument records, our goal is to explore general subglacial behaviors under typical conditions. We thus take a simplified view of the subglacial environment, in which the glacier bed is classified into three categories: (1) soft-bedded regions that are hydraulically-connected to the subglacial drainage system, (2) soft-bedded but poorly-connected regions and (3) hard-bedded regions. Only general assumptions are made about the distribution of bed types (discussed below), and each region is assumed to cover an areal fraction α_i of the bed such that $\alpha_1 + \alpha_2 + \alpha_3 = 1$.

4.2. Pore-water pressure evolution

Subglacial sediment in the two soft-bedded columns is considered to be a fully-saturated two-component mixture of water and solid particle matrix. The expression used to calculate the time-evolution of pore-water pressure within the till layer was developed assuming that (1) the equations governing till porosity and dilatancy equations follow those of *Clarke* [1987], (2) hydraulic permeability is determined by the Kozeny-Carman relation (*Carman*, [1961]), and (3) water flux within the till columns obeys Darcy's law.

Typically, modeling of water flow through a porous medium is accomplished by writing the equations of state in terms of an Eulerian, or spatially fixed, coordinate system. While this approach is appropriate for cases in which deformation of the medium is negligible, a dilatant till can undergo significant volume changes. We therefore use a Lagrangian approach, in which the equations of state are expressed in terms of the initial configuration of particles. In such a coordinate system, Z represents the initial height of a particle in the till column; at some later time t its position is $z(Z,t)$. The Lagrangian equation for the time-evolution of pore-water pressure within the till is

$$\begin{aligned} \tfrac{\partial}{\partial t} p(Z,t) \; = & \left\{ \frac{1}{\mu} \left[\frac{d\kappa}{dp} + \beta\kappa(p) \right] \left[\frac{1}{J(t)} \frac{\partial p}{\partial Z} + \rho_{\rm w} g \right] \frac{\partial p}{\partial Z} \right. \\ & \left. - \frac{\kappa(p)}{\mu\big(1 - n(Z,0)\big)} \frac{dn(p)}{dp} \left(\frac{\partial p}{\partial Z} \right)^2 + \frac{\kappa(p)}{\mu} \frac{1}{J(t)} \frac{\partial^2 p}{\partial Z^2} \right\} \\ & \times \left\{ \big(\alpha(p) + n(Z,t)\beta \big) J(t) \right\}^{-1}. \end{aligned} \tag{7}$$

Here p is the pore-water pressure and ρ_w, μ and β are the density, viscosity and compressibility of water. The terms n, κ, and α represent the porosity, hydraulic permeability, and compressibility of the solid matrix. The acceleration due to gravity is g, and J is the Jacobian of the transformation from Z to z. See Appendix A for a more thorough discussion of the Lagrangian representation and a detailed discussion of the development of this equation.

Dilatation of subglacial sediments results in changes in porosity n and therefore the hydraulic permeability κ. The time-evolution of the porosity is related to the rate-of-change in pore-water pressure, the local strain rate within the till layer, and the consolidation state by

$$\frac{\partial n}{\partial t} = \begin{cases} \dfrac{1}{(1+e)^2} \left[\dfrac{1}{B_s(p' + p_1')} \dot{p} - D_0 \dot{\varepsilon} (e - e_{CS}) \right] & \text{left of NCL} \\ \dfrac{1}{(1+e)^2} \left[\dfrac{1}{B_c(p' + p_1')} \dot{p} - D_0 \dot{\varepsilon} (e - e_{CS}) \right] & \text{on NCL.} \end{cases} \tag{8}$$

Here e is the void ratio at a given depth within the till column and e_{CS} the critical-state void ratio. The term B_s is a swelling index and B_c a compression index. The rate-of-change of the porewater pressure is given by $\dot{p}$ and D_0 is a scaling factor. The term p'_1 prevents the void ratio from diverging as the effective pressure value approaches zero, This equation is developed fully in Appendix A.

4.3. Deformation profile

Assuming simple shear in the till, the strain rate is given by

$$\dot{\varepsilon} = \frac{1}{2J(t)} \frac{\partial v_x}{\partial Z}. \tag{9}$$

Solving (9) for $\partial v_x / \partial Z$ and integrating up from the base of the till layer yields the velocity profile $v(Z, t)$:

$$v_x(Z, t) = 2 \int_0^Z J(t)\, \dot{\varepsilon}(Z', t)\, dZ' \tag{10}$$

Depending on the flow law chosen, the strain rate $\dot{\varepsilon}(Z, t)$ is determined by equation (1), (2), (3) or (6). We assume that no slip occurs between the till base and the underlying substrate. Till displacement is calculated by integrating the velocity field with respect to time:

$$\frac{\partial}{\partial t} s(Z, t) = v_x(Z, t) \tag{11}$$

4.4. Glacier sliding

In addition to facilitating the deformation of subglacial sediments, increased subglacial water pressure also enables basal sliding by submerging roughness features and decoupling the ice from the bed. Several models have been developed to relate glacier sliding to basal shear stress, effective pressure and bed roughness [e.g., *Weertman*, 1957, 1964; *Lliboutry*, 1968, 1987; *Kamb*, 1970]. While these models assume the glacier bed to be rigid and impermeable, the regions of interest in the model developed here are underlain by soft, deforming sediments. Here we use an alternate sliding model, in which the sliding velocity is related to the driving stress τ acting upon the region by

$$v_{SL}(t) = C_{SL}(p) \frac{h_{SL}}{\mu_{SL}} \tau(t). \tag{12}$$

The sliding coefficient C_{SL} is assumed to be related to the water pressure at the ice–bed interface:

$$C_{SL}(p) = \frac{1}{2} \left[1 + \tanh \left(2\pi \frac{p(h_0, t) - p_{SL}}{\Delta p_{SL}} \right) \right]. \tag{13}$$

The value of C_{SL} varies between zero and unity depending on the water pressure at the ice–bed interface. At water pressures much greater than the reference pressure p_{SL} in (13), $C_{SL} = 1$ and the sliding velocity is $v_{SL} = (h_{SL}/\mu_{SL})\tau$, equal to the rate at which a layer of lubricating material of thickness h_{SL} and viscosity μ_{SL} would deform when subjected to shear traction τ. At such pressures, roughness features of the bed are assumed to be completely submerged and the shear stress is transferred to the bed by viscous coupling through the layer of lubricating slurry. At water pressures significantly lower than the reference pressure, sliding is assumed to be negligible because of the strong mechanical coupling between the ice and bed provided by roughness features. The transition between non-sliding and sliding modes is assumed to occur over the pressure range $p_{SL} \pm \Delta p_{SL}/2$. In this transition zone, partial submergence of roughness features reduces their effectiveness and allows increased sliding, and shear traction is assumed to be transferred to the bed by a combination of mechanical resistance and viscous coupling. The sliding velocity v_{SL} is given by the rate-of-change of the sliding distance $s_{SL}(t)$,

$$\frac{\partial}{\partial t} s_{SL}(t) = v_{SL}(t) = C_{SL}(p(h(0), t)) \frac{h_{SL}}{\mu_{SL}} \tau(t). \tag{14}$$

The total ice velocity and displacement in each column are simply the sum of the deformational and sliding components:

$$V(t) = v(h(0), t) + v_{SL}(t) \tag{15}$$

$$S(t) = s(h(0), t) + s_{SL}(t). \tag{16}$$

4.5. Ice-dynamics model and basal shear stress

For a glacier with thickness H_I and surface slope θ, the nominal shear stress acting upon the bed is $\tau_0 = \rho_I g H_I \sin\theta$ (Fig. 3). The local shear stress can vary from this nominal value if basal conditions favor the transfer of stress to or from neighboring regions. Because subglacial water pressure strongly influences both sediment strength and basal sliding, diurnal pressure variations can drive a cyclic transfer of shear stress between regions of the bed. Although the transfer of shear stress between basal regions can result in unequal stress distribution, the area-averaged shear stress is equal to the nominal value. In our three-column representation, this is expressed

$$\alpha_1 \tau_1 + \alpha_2 \tau_2 + \alpha_3 \tau_3 = \tau_0, \tag{17}$$

where α_1, α_2 and α_3 are assumed fixed. It follows that

$$\alpha_1 \frac{\partial}{\partial t}\tau_1 + \alpha_2 \frac{\partial}{\partial t}\tau_2 + \alpha_3 \frac{\partial}{\partial t}\tau_3 = 0. \tag{18}$$

A rigorous treatment of the transfer of stresses between regions would require computation of the full three-dimensional stress and velocity fields within the ice. Although such algorithms have been developed [e.g., *Blatter*, 1995], they add greatly to the computational complexity and require knowledge of the distribution of basal stresses. We instead employ a simple ice-dynamics model adapted from *Fischer and Clarke* [1997] that allows investigation of the effects of stress bridging while retaining computational simplicity. We assume that the columns representing the different basal regions are located across-slope (*i.e.*, aligned perpendicularly to the flow direction; Fig. 5). Soft-bedded regions are assumed to be separated from each other by a characteristic distance L_S; hard-bedded and soft-bedded regions are separated by distance L_H. This geometry allows us to calculate stress bridging in terms of simple shear of the ice between columns. Ice is treated as a nonlinear Maxwell material:

$$\dot{\varepsilon} = \frac{1}{2G_I} \dot{\tau} + A_0 \tau^N. \tag{19}$$

The short-term elastic response is determined by the rigidity G_I, and, for $N = 3$, the material deforms over long timescales according to Glen's Law [*Glen*, 1958]. If we designate the shear stress transferred from column b to column a as $\tau_{b \to a}$, rearranging (19) yields

$$\dot{\tau}_{b \to a} = 2G_I \left(\dot{\varepsilon}_{b \to a} - A_0 \tau_{b \to a}^N \right). \quad (20)$$

Assuming uniform simple shear of the ice between columns gives

$$\dot{\varepsilon}_{b \to a} = \dot{\varepsilon}_{xy} = \frac{1}{2}\frac{\partial v_x}{\partial y} = \frac{\Delta V}{2L}, \quad (21)$$

where $\Delta V = V_b - V_a$ is the difference in ice velocity between columns and L is the characteristic distance separating the columns. Depending on which pair of columns are being considered, L is replaced by either L_S or L_H. Substitution of (21) into (20) yields

$$\dot{\tau}_{b \to a} = 2G_I \left(\frac{\Delta V}{2L} - A_0 \tau_{b \to a}^N \right). \quad (22)$$

The preceding discussion considers stress bridging between one pair of columns. Full accounting of stress bridging takes into account the transfers between each pair of columns and the relative areal coverage of each column in a manner that satisfies equation (18):

$$\begin{aligned} \frac{\partial}{\partial t}\tau_1 &= \dot{\tau}_{2 \to 1} + \dot{\tau}_{3 \to 1} \quad (23) \\ \frac{\partial}{\partial t}\tau_2 &= -\frac{\alpha_1}{\alpha_2}\dot{\tau}_{2 \to 1} + \dot{\tau}_{3 \to 2} \\ \frac{\partial}{\partial t}\tau_3 &= -\frac{\alpha_1}{\alpha_3}\dot{\tau}_{3 \to 1} - \frac{\alpha_2}{\alpha_3}\dot{\tau}_{3 \to 2}. \end{aligned}$$

Values for $\dot{\tau}_{2 \to 1}$, $\dot{\tau}_{3 \to 1}$ and $\dot{\tau}_{3 \to 2}$ are calculated by substituting appropriate values for velocity contrast ΔV, separation distance L and transferred stress $\tau_{b \to a}$ for each pair of columns into (22). The shear stress acting upon the till in a given region is assumed constant throughout the layer (i.e. no z-variation; *Alley* [1989] has called this the "thin till approximation"). For simplicity we further neglect any vertical velocity gradient in the ice, and flow mechanisms such as enhanced creep of the basal ice, regelation, and ploughing are ignored.

This ice-dynamics model assumes uniform simple shear of ice between basal regions. Uniform deformation is likely to occur only if basal conditions are homogeneous across the bed separating the columns, as changes in basal conditions would tend to localize deformation. *Blatter et al.* [1998] modeled the stress and velocity conditions in the vicinity of an isolated sliding spot in an otherwise homogeneous non-sliding slab. Their modeling results suggest that the surface velocity distribution would vary smoothly over lengthscales equivalent to several ice thicknesses. The assumption of uniform simple shear between columns is therefore likely a reasonable simplification.

4.6. Synthetic instrument responses

Instrument responses calculated from the modeled pore-water pressure, till deformation, glacier sliding, and till strength conditions. Because all of the records shown in Figure (2) were recorded by instruments installed in a hydraulically-connected region of the bed, and because no suitable records exist for the unconnected region, comparison between synthetic and field instrument responses will be limited to the connected region of the bed. Modeled instruments include pressure transducers, ploughmeters, bed tilt sensors, and slidometers. In the field, these instruments are hammered ~0.05–0.25 m into the subglacial till depending on instrument type and local bed properties. The true depth of installation is uncertain, however, as (1) hot-water drilling disturbs the sediments at the base of the borehole and (2) it is possible for soft subglacial sediments to ooze up into the borehole. Because variations in installation depth influences the instrument response, we calculate instrument responses for a range of installation depths.

4.6.1. Bed tilt

Tilt cells [*Blake et al.*, 1992] measure deformation of the subglacial till. Upon installation the cell is oriented approximately vertically, and any deformation of till in the range of depth occupied by the sensor results in rotation of the device.

Assuming simple shear of the sediments, the average strain rate in the depth range occupied by the tilt cell is given by

$$\dot{\varepsilon} = \frac{1}{2}\frac{\partial \tan\theta_{BT}}{\partial t}. \quad (24)$$

We assume that the tilt cell is advected passively with the deforming till. The bottom of the instrument is thus located at $[x_b(t), z_b(t)] = [s(Z_0, t), Z_0]$, where Z_0 is the installation depth, and the top of the tilt cell is coincident with the point on the deformation profile $s(Z, t)$ distance l_{BT} (the device length) away from $[x_b(t), z_b(t)]$. The tilt angle is given by

$$\theta_{BT} = \tan^{-1}\left(\frac{z_t - z_b}{x_t - x_b} \right). \quad (25)$$

Because the Eulerian coordinates z_b and z_t used in determining the tilt angle are calculated from the Lagrangian representation of the till matrix, rotation of the device due to dilatant expansion and contraction of the till is incorporated into (25).

4.6.2. Pressure transducer

Small pressure transducers are usually incorporated into the tip of the tilt cell housing, allowing simultaneous measurement of basal deformation and pore-water pressure. The pore-water pressure is thus

$$p_{PZ}(t) = p(Z_0, t) \quad (26)$$

where Z_0 is the installation depth of the tilt cell.

4.6.3. Slidometer

Slidometers ([*Blake et al.*, 1994]) consist of a thin line, a spool (onto which the line is wound and that turns a small potentiometer as the glacier slides over the bed), and an anchor (which attaches the line to the bed). Ideally, the measured displacement would result solely from glacier sliding and would be unaffected by basal deformation. However, because it is necessary to firmly affix the line to the bed, the anchor is placed some distance into the till. The displacement signal is therefore contaminated by any sediment deformation that occurs above the anchor placement. Assuming that the string connecting does not cut through the till and no slippage of the anchor occurs, the total displacement indicated by the slidometer is

$$x_{SL}(t) = s_{SL}(t) + \left\{ \int_{Z_0}^{h(0)} J(t) \left[1 + \left(\frac{\partial x(Z,t)}{\partial Z} \right)^2 \right]^{\frac{1}{2}} dZ \quad - Z_0 \right\}, \quad (27)$$

where Z_0 is the installation depth of the anchor. The true sliding distance is $s_{SL}(t)$ and the second term on the right-hand-side of is the component contributed by sediment deformation.

4.6.4. Ploughmeter

The ploughmeter [*Fischer and Clarke*, 1994] consists of a 1.54 m steel rod that is instrumented with strain gauges, and is installed so that the tip is embedded ~0.1–0.2 m into the subglacial sediments. Relative motion between the sediments and the ploughmeter flexes the steel rod, and this flex is measured by the strain gauges. Ploughmeter calibrations are typically performed by hanging objects of known weight from the tip, and as a result calibrations (and subsequent field records) are commonly expressed in units of force (kN) rather than of bending moment (N m). To allow direct comparison with field records, we divide the modeled bending moment M by the tip-to-gauge-center distance l_g. Assuming that that the drag force is acting perpendicularly to the ploughmeter axis, the equivalent force acting upon the ploughmeter tip is thus

$$F = \frac{M}{l_g} = \frac{1}{l_g} \int_{Z_0}^{h_g} [h_g - Z]\, J(t)\, F'(Z)\, dZ, \qquad (28)$$

where $F'(Z)$ is the force per unit length applied between Z and $Z + dZ$.

Following [*Fischer and Clarke*, 1994], the force per unit length for flow laws that have linear or nearly-linear dependencies on *tau* is calculated as

$$F' = \frac{4\pi\eta(Z,t)\Delta V(Z,t)}{\ln(2l_g/r_{PL}) + 0.5} \qquad (29)$$

[*Batchelor*, 1970; *Cox*, 1970; *Tillett*, 1970]. Here $r_{PL} = 0.015$ m is the ploughmeter radius. The force acting upon the ploughmeter by a viscous fluid is proportional to both the velocity contrast $\Delta V(Z,t) = V(t) - v(Z,t)$ between the ploughmeter and sediments at depth Z and the effective viscosity η at that depth, given by

$$\eta(Z,t) = \frac{1}{2\dot{\varepsilon}(Z,t)}\,\tau(t). \qquad (30)$$

This term incorporates any any stress- and pressure-dependent nonlinearities in flow; for linear-viscous tills $\eta(Z,t) = \eta_0$. The small degree of nonlinearity suggested by the values of a and b suggested by *Boulton and Hindmarsh* [1987] for (2) (at 1.33 and 1.80) and (3) (at 0.625 and 1.25) make this a reasonable approximation.

Following the analysis of *Humphrey et al.* [1993] and *Fischer and Clarke* [1994], the total force per unit length acting upon a ploughmeter moving through a Coulomb-plastic till is

$$F'(Z,t) = 4r_{PL}(2+\pi)\sigma_Y(Z,t), \qquad (31)$$

where the yield stress σ_Y is given by the Mohr-Coulomb failure criterion (4). Because the yield strength of Coulomb-plastic till is independent of the strain rate, the velocity contrast between the ploughmeter and till does not appear in (31).

The total force acting upon a ploughmeter due to motion through a till is given by substituting (29) for linear-viscous or nonlinear-viscous tills or (31) for Coulomb-plastic tills into (28). For nonlinear-Bingham tills, this substitution depends on the value of the shear stress. At stresses below σ_Y, elastic behavior dominates, and we assume that the force acting on the ploughmeter is given by equation (31). At high basal shear stresses, viscous behavior dominates, and (29) is used. At intermediate shear stress values, till behavior is neither truly elastic nor truly viscous. For given values of a and σ_Y in (3), the minimum effective viscosity occurs at $\sigma_{min} = \sigma_Y/(1-a)$. In the range of values $\sigma_Y < \tau < \sigma_{min}$ (Fig. 4c, shaded region), the bending moment is therefore taken to be the greater of the values calculated using (29) and (31).

5. Model parameters

Table 1 shows values for physical constants used in the model. Values used for parameters G_I and A_0 are appropriate for glacier ice at $-5°$C, which represents an approximate depth-averaged temperature value for Trapridge Glacier. In the ice-dynamics model developed above, it is the quantity $A_0 L$, where L is the characteristic lengthscale between columns, that determines ice behavior. We will thus consider A_0 fixed and take L as the adjustable parameter.

The geotechnical properties of Trapridge Glacier till are not currently well known. Size-distribution analysis of a sample of recently-exposed basal till from Trapridge Glacier showed that fine-grained solids (silt- and clay-sized particles) account for roughly 40% of the solid volume [*Clarke*, 1987]. Other quantities, such as the *in situ* porosity, compressibility, permeability and shear strength, are poorly constrained. We take parameter values pertaining to these properties from previous instrument studies at Trapridge Glacier or from geotechnical tests of tills obtained from other locations.

Assumed till property parameter values used in the model are shown in Table 1. The value chosen for the sediment density $\rho_s = 2800\ \mathrm{kg\,m^{-3}}$ is representative of the mix of metamorphic and igneous materials found beneath Trapridge Glacier. Reference values chosen for the normal consolidation void ratios e_0 and e_1 and for the critical-state reference void ratio e_{CS} are slightly lower than those used by *Clarke* [1987] and are likely more representative of Trapridge till, yielding initial porosities of $0.21 \le n \le 0.28$ for the range of effective pressures encountered beneath Trapridge Glacier. Assuming a surface-to-volume ratio of $S_0 = 5.0 \times 10^6\ \mathrm{m^{-1}}$ (corresponding to a clay-sized particle with grain diameter 1.2 μm) results in hydraulic permeabilities of $k = 1.19$–$3.39 \times 10^{-16}\ \mathrm{m^2}$, reasonable values for till. The rate of dilatant till expansion or contraction is determined by the scaling factor $A_{CR} = 0.20$; this value results in a time constant of $\dot{\varepsilon}/5$.

Table 2 shows the assumed parameter values for the four modeled till flow laws. The viscosity chosen for the linear-viscous flow law is taken from earlier instrument studies performed at Trapridge Glacier. For the nonlinear-viscous and nonlinear-Bingham flow laws, we adopt the values for B_2, a and b in (2) and (3) proposed by *Boulton and Hindmarsh* [1987], but use a value for B_1 in (2) that is approximately 30× greater than that proposed by *Boulton and Hindmarsh* [1987] in order to produce deformation rates closer to those measured at Trapridge Glacier.

Both nonlinear-Bingham and Coulomb-plastic tills deform only at shear stresses greater than a yield stress determined by the Mohr-Coulomb failure criterion. To date, no shear strength measurements have been performed on

Trapridge Glacier till, and thus we use values for c_0 and ϕ measured for a clay-rich till by *Iverson et al.* [1998].

Shear failure of an ideal plastic material would be localized to an infinitely thin layer. However, because till is composed of particles with sizes ranging from clays (diameter $<1\,\mu$m) to boulders (diameter >1 m), till deformation will be distributed over a layer with finite thickness. The values we have chosen for the reference strain rate $\dot{\varepsilon}_0 = 1.0\times10^5\,\mathrm{s}^{-1}$ and failure range $\Delta\tau = 5.0$ kPa in (6) result in deformation that is distributed over a layer up to 0.24 m in thickness, comparable in scale to the diameter of the largest commonly-occurring clasts.

Clearly, model results are strongly influenced by the choice of till flow law parameters. We discuss the effects of varying the flow law parameters in Appendix B.

5.1. Boundary conditions and initial conditions

Table 3 lists values prescribed to boundary and initial conditions for the model results presented in the following sections. The main "tuning" parameters in the model include the water pressure at the base of the till layer p_B, till layer thickness h, length scales L_H and L_S, areal fractions α_i, and glacier sliding reference pressure p_{SL}. The effects of varying these key parameters are discussed in Appendix B.

Values for ice thickness $H_I = 70.0$ m and surface slope $\theta = 7.0°$ were chosen to represent the geometry of Trapridge Glacier, and the thickness of the deformable till layer is assumed to be 1.0 m (~3 times greater than the depth of deformation inferred from field studies [*Blake*, 1992]). Appropriate values for areal fractions α_1, α_2 and α_3 are not well established. In the study area, attempts to install instruments in the bed are usually successful, suggesting that the study area is largely soft-bedded. Crevasse patterns in the vicinity of the study area suggest that resistance to flow could be provided mainly by marginal drag. We make the rough assumptions that (1) hydraulic connection occurs in 25% of soft-bedded regions and (2) hard-bedded and hydraulically-connected regions cover approximately equal portions of the bed. These assumptions give nominal values $\alpha_1 = 0.20$, $\alpha_2 = 0.60$ and $\alpha_3 = 0.20$.

Water pressure values are prescribed at the ice–bed interface in the two soft-bedded columns. In order to model general summer behaviors at Trapridge Glacier, we choose pressure forcing functions for columns 1 and 2 that mimic typical summer-mode pressure records in connected and unconnected regions, respectively. Hydraulic connection in column 1 is simulated by assigning a sinusoidally-varying pressure function to the top of the till column:

$$p_1(h,t) = p_{T1} + \Delta p_{T1}\cos(\omega t). \tag{32}$$

The pressure $p_1(h,t)$ thus represents the drainage system pressure. Taking $p_{T1} = 490.0$ kPa, $\Delta p_{T1} = 98.0$ kPa and $\omega = 2\pi\,\mathrm{d}^{-1}$ yields diurnally-varying pressures ranging between 40.0 and 60.0 m of head, similar to the record for P1 (Fig. 2a). Unconnected regions generally exhibit steady, near-flotation pressure values, and so the water pressure at the top of column 2 is $p_2(h,t) = p_{T2} = 588.0$ kPa (equivalent to 60.0 m of head).

The water pressure at the base of the till layer is poorly constrained. If the region beneath the till layer is relatively impermeable, p_B would reflect a long-term average of pressures at the ice–bed-interface. If the till layer is underlain by a highly permeable aquifer, however, p_B may differ from this value depending upon the aquifer pressure. For the modeling scenarios presented here, we assign a nominal basal pore-water pressure of $p_B = 294.0$ kPa (30.0 m of head) to both till columns. The initial pore-water pressure profile within the till layer for each of the two soft-bedded columns is assumed to vary linearly with depth between the values prescribed at the top and bottom of the till layer:

$$p(Z,0) = p_B + (p_T - p_B)(h - Z)/h. \tag{33}$$

Void ratio values for each column are initialized to the critical state value (A10) given the assumed pressure profiles:

$$e(Z,0) = e_{CS}(p_I - p(Z,0)). \tag{34}$$

In the majority of the modeling scenarios presented here, we assume that glacier sliding occurs only in the hydraulically-connected regions of the bed, even though modeled water pressures in the hydraulically-unconnected region are near flotation. This assumption, which is valid if the volume of free water in unconnected regions is insufficient to submerge roughness elements of the bed, has been made primarily to prevent pervasive ice–bed decoupling. We discuss relaxation of this assumption in Appendix B.

The sliding parameters in (13) assigned to column 1, at $p_{SL} = 698.5$ kPa and $\Delta p_{SL} = 629.1$ kPa $(= p_I)$, result in 20% decoupling at flotation pressure ($p = p_I$) and ~7% decoupling at $p = 0.90\,p_I$. This value of p_{SL} was chosen to yield sliding velocities similar to those recorded by SL1 (Fig. 2). We assume that the lubricating slurry has viscosity $\eta_{SL} = 2.0\times10^7$ Pa s (0.1% of the linear till viscosity) and a layer thickness of 0.01 m.

In nature, glacier motion over hard-bedded regions is due to some combination of creep flow, ice fracture and basal sliding. These mechanisms are ignored in the simple ice-dynamics model employed here; ice above column 3 is assumed static and provides necessary resistance to glacier flow. The value $L_H = 1500$ m chosen for the lengthscale of variation between hard- and soft-bedded regions yields flow rates comparable to those observed at Trapridge Glacier during the summer field season (~0.07–0.12 m d^{-1}). This value is unrealistic (as it exceeds the glacier width at the elevation of the study area), but accounts for the contributions to ice motion made by the mechanisms of enhanced creep, regelation and crevassing not directly incorporated into the ice-dynamics model. Making the rough assumption that $L_S \approx L_H/10$ yields $L_S = 150.0$ m.

6. Modeling results

6.1. Pore-water and deformation profiles

In nature, meltwater-driven changes in drainage system pressure lead to changes in porewater pressure, sediment strength, and glacier sliding, changes that are reflected in instrument records such as those shown in Figure 2. Similarly, all variations in the modeling results presented originate from the prescribed water pressure function (32) at ice–bed interface in the connected region. In the following discussions, we will refer to this pressure as the "modeled drainage system pressure". Figures 6a–d show modeled pore-water pressure profiles in the connected region at 0.125 d intervals. Pore-water pressures at the time of maximum system pressure are shown in Figure 6a (solid line). Pressure profiles during times of decreasing system pressures are shown in Figures 6a, b; Figures 6c, d show pressures increasing from the daily minimum value of 40.0 m. The diurnal pressure signal is seen to diffuse into the till layer to a depth of ~0.5 m. Figure 6e shows the pressure profile for the unconnected region.

Modeled till deformation profiles are shown in Figures 6f–y. Linear-viscous (Fig. 6f–j) and nonlinear-viscous (Fig. 6k–o) tills both exhibit deformation over the full till layer thickness. Because the strength of Linear-viscous till is independent of pore-water pressure, uniform deformation is seen with depth at all pressures. In contrast, the strength of nonlinear-viscous till (Fig. 6k–o) depends on the pore-water pressure. During times of high modeled system pressure, elevated pore-water pressures within the top layer of till, weakening the sediments and concentrating deformation there.

Nonlinear-Bingham (Fig. 6p–t) till shows deformation that is limited to the upper portion of the till layer. Because pore-water pressures generally decrease with depth in the till layer, the resulting high Mohr-Coulomb yield strengths prohibit deformation at depth. As a result, deformation is confined to the top ~0.4 m of the till layer in the connected region and to the upper 0.15 m in the unconnected region.

Deformation of Coulomb-plastic till (Fig. 6u–y) occurs only where pore-water pressures are greatest in the till column. This behavior results in a diurnal migration of the depth of active deformation. At the time of minimum system pressure, the deforming region is centred at a depth of ~0.27 m (Fig. 6w, solid line), corresponding to the depth of maximum pore-water pressure (Fig. 6c, solid line). Three hours later, when the maximum pore-water pressure is again found at the ice–bed interface (Fig. 6w, dashed line), deformation is confined to the top of the till layer.

Modeled deformation of Coulomb-plastic till occurs over a localized region ranging in thickness from 0.01 m at maximum system pressure to 0.24 m at minimum system pressure. These variations in deforming-layer thickness result from our treatment of plastic deformation, in which failure is assumed to occur over some stress range $\Delta\tau$ in (6). Because the yield stress varies with pore-water pressure, steep pressure gradients in the till (as seen at maximum pressure, Fig. 6a, solid line) result in an abrupt transition between deforming and non-deforming states, with deformation confined to the top of the till layer. Lower pressure gradients, as seen at the time of minimum system pressure (Fig. 6c, solid line), yield a more gradual transition.

Table 4 shows sediment deformation, glacier sliding, and total basal motion rates for the connected region at times of maximum and minimum modeled drainage system pressure, along with sediment deformation rates at these times in the unconnected region (as sliding is prohibited there). For all flow laws, ice–bed decoupling at high system pressures results in reduced shear traction acting upon the bed in the connected region, decreasing deformation rates and increasing sliding rates there. This table also shows that the difference in total flow rate between times of maximum and minimum system pressure increases with increasing flow-law nonlinearity. Because flow-rate measurements at Trapridge Glacier are only made on a daily bases, it is unclear how the ice velocity changes over the course of a diurnal pressure cycle. It would be interesting to investigate what the magnitude of any such changes might say about the rheological nature of till.

Figure 7 shows total daily sediment deformation in the connected (Fig. 7a–d) and unconnected (Fig. 7e–h) regions for each of the four till flow laws. In all cases the total deformation is greater in the unconnected region because (1) glacier sliding accounts for a portion of glacier motion in the connected region and (2) for nonlinear flow laws, stiffening of till in the connected region during times of low system pressure reduces the deformation rate there. Linear-viscous till (Fig. 7a, e) exhibits uniform deformation over the full till layer, while for the other flow laws deformation is increasingly concentrated at the ice–bed interface with increasing flow-law nonlinearity. This is most evident in the unconnected region: for nonlinear viscous till, 74% of deformation occurs in the top 0.25 m of the till layer, whereas deformation of nonlinear-Bingham and Coulomb-plastic tills is limited to the top 0.18 m and 0.03 m, respectively. Deformation of nonlinear-Bingham and Coulomb-plastic tills extends to greater depths in the connected region (Fig. 7c, inset; 12d) due to diffusion of pressure variations, reaching depths of 0.45 and 0.34 m.

6.2. Basal shear stress

Figure 8 shows modeled five-day records of basal shear stress in each region for the four till flow laws; the modeled drainage system pressure is shown for reference. For all flow laws, the shear stress is predominantly concentrated on the hard-bedded region, which supports an average of $4\tau_0$. Also evident in all four cases is the water-pressure driven transfer of shear stress between regions. During times of high pressure, stress is transferred from the connected region to the unconnected and hard-bedded regions by enhanced deformation and glacier sliding. Drainage system pressure and connected-region shear-stress signals are thus out-of-phase, while stresses in the unconnected and hard-bedded regions show nearly in-phase responses to drainage system pressure changes.

The minimum and maximum shear stresses supported in each region are listed in Table 5. Hard-bedded regions provide the main resistance to ice motion for all flow laws, and shear stresses supported by soft-bedded regions for all flow laws are similar to estimates of basal stress for Ice Stream B by *Kamb* [1991] and *Tulaczyk et al.* [2000c]. These studies suggest that basal drag accounts for ~10–50% of the total flow resistance. Linear-viscous tills exhibit relatively minor redistribution of basal stresses with variations in drainage system pressure, as the deformation rate for this flow law is independent of pore-water pressure and any stress transfer is due to changes in sliding velocity. Nonlinear-viscous and Bingham tills exhibit more pronounced changes in stress distribution, as till stiffening in the connected region during times of low system pressure results in the transfer of shear stress onto that region from the connected and hard-bedded regions. While a similar exchange of stresses occurs between the connected and hard-bedded regions for Coulomb-plastic till, the stress supported by the unconnected region remains constant as a result of the rate-independent nature of this flow law.

6.3. Dilatant response

Dilatant response to diurnal pressure fluctuations result in variations in till column thickness of 1.6 mm for all flow laws. This small response is due to the fact that void ratio values in this modeling scenario are close to the critical state value given by (A10). In situations where void ratios differ substantially from the critical-state value, dilatant properties may significantly affect till behavior. Thus while the modeling results presented here show only minor dilatant responses, their inclusion could prove important in modeling such scenarios as the onset of deformation in a highly-overconsolidated till. For this reason, we believe it is essential to retain the Lagrangian representation of till despite the minor dilatant response noted here.

6.4. Modeled instrument responses

6.4.1. Pressure transducer

Records for field and modeled pressure transducers are shown in Figure 9. The pressure records for P1 (solid line) and PZ1 (dotted line) are shown in Figure 9a; modeled responses shown in Figure 9b. The solid line in Figure 9b represents the prescribed pressure function at the ice–bed interface. Modeled pore-water pressures at depths of 0.15 (dotted line), 0.25 (dashed line) and 0.35 m (dot-dashed line) for linear-viscous till are also shown. The other till flow laws produced similar pressure profiles, with pore-water pressures differing by less than 0.025 m in all cases. These small differences are due to slight dilatancy-driven variations in permeability structure. Modeled drainage-system and pore-water pressures show a relationship similar to that between P1 and PZ1, with pressures within the till layer exhibiting variations that are smaller in amplitude and lagged in phase in comparison with those in the drainage system. The record for PZ1 shows peak-to-peak pressure variations 44–78% of those indicated by P1 and exhibit lags of 40–86 min. Modeled pore-water pressure variations range between 33 and 61% of the 20.0 m diurnal pressure variation and lag by 106–252 min. Although modeled and field records at the estimated installation depth for PZ1 (0.15 m) show reasonable agreement, the lower amplitudes and greater lags indicated by synthetic records suggests that modeled permeabilities are too low.

6.4.2. Tilt cell

Figure 10 shows till strain rates calculated from field (Fig. 10a) and modeled (Fig. 10b–e) tilt cell records. Modeled responses are calculated for installation depths of 0.15 (solid line), 0.25 (dotted line) and 0.35 m (dashed line). All synthetic records indicate minimum deformation rates at times of high system pressure due to ice–bed decoupling during these times. This behavior is also seen in the record for BT1 (Fig. 10a). Because deformation of linear-viscous till (Fig. 10b) is uniform over the full till depth, similar responses are reported for the three modeled installation depths. Although the response for linear-viscous till is of similar form to that for BT1, the indicated strain rate is an order of magnitude lower, ranging between 0.021 and 0.044 d^{-1}. Deformation rates for nonlinear-viscous till (Fig. 10b) vary with installation depth, with higher strain rates seen nearer the top of the till layer. Till stiffening at low system pressures results in a slight reduction in deformation rate during these times. Indicated peak deformation rates of 0.033–0.041 d^{-1} are 6–14% those indicated by BT1.

Records for nonlinear-Bingham and Coulomb-plastic tills (Fig. 10d, e) indicate no deformation at high system pressures. The stiffness of the Bingham till dictates that sliding dominates at these times; for Coulomb-plastic till, deformation is confined to a thin region above the tilt cell. The calculated strain rate for both records show strong dependence on installation depth, with peak strain rates occurring later with deeper installation. Peak deformation rates for Bingham till are 0.022–0.028 d^{-1}, ~4–10% of those indicated by BT1. Coulomb-plastic till exhibits higher deformation rates, with maximum values of 0.12–0.16 d^{-1}. The twin strain-rate peaks at installation depths of 0.15 and 0.25 m arise from cyclic migration of the deforming region with system pressure variations. Slightly negative strain rates indicated in the responses for Bingham and Coulomb-plastic tills at peak system pressure is due to dilatant expansion of the till.

6.4.3. Slidometer

Figure 11 shows glacier sliding rates determined from field (Fig. 11a) and modeled (Fig. 11b–e) slidometer records. Responses are calculated for installation depths of 0.10 (dotted line), 0.20 (dashed line) and 0.30 m (dot-dashed line), and the true modeled sliding rate is also shown (solid line). Modeled records show sliding rates and phase relations similar to those indicated during days 202.5–204.5 in the record for SL1 (Fig 16a). Peak sliding rates indicated by SL1 during this two-day period are 0.072 and 0.079 m d^{-1}. Peak indicated sliding rates for linear-viscous, nonlinear-viscous and Bingham tills (Fig. 11b–d) are close to the true value for all installation depths, with deformation adding ~0.004–0.007 m d^{-1} to the true value. Coulomb-plastic till exhibit significant deformation in the uppermost portion of the till layer at times of rising system pressure, a behavior that, while not seen in the record for SL1, has been observed in sliding records for Trapridge Glacier [*Blake et al.*, 1994; *Fischer and Clarke*, 1997]. Indicated sliding rates at these times are approximately double the true sliding rate (Fig. 11e). Peak indicated sliding rates for linear-viscous, nonlinear-viscous, nonlinear-Bingham and Coulomb-plastic tills are 0.051, 0.066, 0.076 and 0.121 m d^{-1}, respectively; minimum sliding rates for the four till flow laws range 0.001–0.008 m d^{-1}.

6.4.4. Ploughmeter

Modeled ploughmeter records are shown in Figure 12b–e, with the record for PL1 shown for comparison (Fig. 12a). Ploughmeter responses are calculated for installation depths of 0.10 m (solid line), 0.20 m (dotted line) and 0.30 m (dashed line), with greater installation depths resulting in higher calculated force values. Figure 12b shows the modeled ploughmeter response for linear-viscous till. Force values are seen to be in-phase with system pressure (thick grey line). Because till properties do not vary with time, variations in the calculated force result from increased sliding velocities at times of high system pressure. Peak force values range between 2.3 and 2.6 kN for the modeled installation depths. Minimum force values, which occur at the time of minimum system pressure (and thus minimum sliding) are between 0.3 and 1.1 kN. Although forces in the record for PL1 (Fig. 12a, solid line) are of similar magnitude to those shown in Figure 12a, peak modeled force values exhibit an in-phase relation with water pressure (grey line), as opposed to the out-of-phase relation seen in the records for PL1 and P1.

Maximum ploughmeter force values for non-linear viscous till (Fig. 12c) occur at times of rising system pressure. At this time, increasing sliding rates drag the ploughmeter through till that is relatively stiff. Diffusion of high pressures down into the till layer results in till softening and reduced ploughmeter force values, with softening occurring at later times for greater installation depths. Stiffening of the till following peak system pressure gives rise to the small increase noted in the 0.10 m response (solid line). Maximum force values range between 4.0 and 12.2 kN, with minimum values of 1.4–2.9 kN. Ploughmeter responses for Bingham till (Fig. 12d) exhibit similar characteristics to those for non-linear viscous till. Neither till flow law reproduces the observed phase relationship between ploughmeter force values and drainage-system pressure seen in records for instruments PL1 and P1 (Fig. 12a). Maximum force values calculated for Bingham till are 4.4–8.4 kN; minimum values are 1.2–1.8 kN

Ploughmeter responses for Coulomb-plastic till (Fig. 12e) are similar in both magnitude and phase to records for field instruments PL1 and P1 (Fig. 12a). Maximum force values occur immediately following the time of minimum system pressure, with magnitudes of 1.12–1.15 kN. Minimum forces range between 0.32 and 0.71 kN and lag peak pressure by 64–236 min due to the low permeability of the till.

7. Discussion

In the following discussion we compare modeled instrument responses with field records in an effort to determine the till flow law that provides the best qualitative match to observed variations in hydrological and mechanical conditions.

Modeled strain rates for linear-viscous and nonlinear-viscous tills are an order of magnitude lower than those indicated by the field record for BT1 (Fig. 10a–c). This is a result of the fact that deformation of these tills occurs over the full till-layer thickness (Fig. 6f–j, k–o). Thus while the total deformation rates for these flow laws tills are relatively high, the strain rate at any given depth is low. For nonlinear-Bingham till, deformation is limited to the upper portion of the till layer (Fig. 6p–t), but the inherent stiffness of this flow law results in peak modeled strain rates that are just 4–10% of those indicated by the record for BT1 (Fig. 10d). Peak modeled strain rates for Coulomb-plastic till are significantly greater than rates calculated for the other flow laws, measuring 24–55% of those indicated by BT1.

The record for BT1 indicates rotation throughout the diurnal pressure cycle. In this regard, the signal for linear-viscous till (Fig. 10b) most closely matches the field record, though at strain rates an order of magnitude lower. Modeled tilt cell rotation for Coulomb-plastic till occurs only during times of low system pressure due to strain localization. A more realistic treatment of strain distribution for Coulomb-plastic till, taking into account grain–grain interactions that would distribute failure over a length scale related to the grain-size distribution, could produce a deformation signal closer in character to that for a linear-viscous till (though at reduced strain rates). Strain rates indicated in the record for BT1 (Fig. 10a) are too high to be representative of rates over a significant layer thickness and are likely indicative of localized deformation.

Modeled slidometer records for all flow relations show similar character to that for SL1 (Fig. 11) and are thus not diagnostic. In contrast, significant differences between flow laws are seen in the phase relationship between modeled ploughmeter force values and drainage system pressures. Because of these differences, comparison of field and modeled ploughmeter responses provides perhaps the clearest means of identifying the flow law that best describes till behavior. The magnitude of computed force values for linear-viscous till are in reasonable agreement with those for PL1 (Fig. 12a, b), but the phase relation with system pressure is opposite to that for PL1. Synthetic ploughmeter responses for nonlinear-viscous and Bingham tills (Fig. 12c, d) are approximately an order of magnitude larger than those for PL1. The ploughmeter force/system pressure phase characteristics also differ, with peak force values occurring at times of rising system pressure. Modeled ploughmeter responses for Coulomb-plastic till show responses that are similar to that exhibited by PL1, both in magnitude and phase with respect to the subglacial drainage system pressure.

These comparisons of field and modeled responses for tilt cells, slidometers and ploughmeters indicate that of the four flow laws investigated, till behavior is best represented by Coulomb-plastic failure. Of the instruments used in this determination, the ploughmeter is perhaps the most diagnostic indicator of till behavior. Only Coulomb-plastic till reproduces the out-of-phase relationship with drainage system pressure commonly observed in field records from Trapridge Glacier. Over the range of model parameter and boundary-condition values that we have investigated, no combination has produced the observed phase relationship for any of the three other flow laws. In contrast, this phase relationship is a robust feature of Coulomb-plastic till, resulting directly from the rate-independence of such materials.

Features in the records for PL1 and SL1 (Fig. 2c, d) further suggest rate-independent behavior. No correlation is seen between ploughmeter force and sliding rate in these records. This is most apparent during days 205–207, during which time the sliding rate varies strongly in a manner unrelated to variations in water pressure (Fig. 2a). The record for PL1, which was installed <1 m away, does not reflect these variations. Similar independence between ploughmeter force values and glacier flow rate have been observed at Storglaciären, Sweden [*Hooke et al.*, 1997]. These characteristics strongly suggest Coulomb-plastic behavior.

It is likely that the simple ice-dynamics model used in these studies greatly oversimplifies the nature of the interactions between various regions of the glacier bed. The across-glacier arrangement of the columns assumed in this model approximates the situation in which boundaries between the basal regions are aligned parallel to the direction of glacier flow. While the similarities between field instrument records from Trapridge Glacier and modeled instrument responses suggest that this approximation is acceptable, it is unclear how the glacier response would differ assuming different configurations of basal regions. For any reasonable basal geometry, however, it is unlikely that two key behaviors would change significantly. First, given the generally weak tills modeled in this study, hard-bedded regions are likely to support a large portion of the driving stress. Second, diurnal variations in the drainage system pressure would result in fluctuations of sediment strength and ice–bed coupling, resulting in diurnal changes in the ice flow rate and the cyclic transfer of shear stress between basal regions. Given these similar characteristics, it is unlikely that instrument responses would differ fundamentally from those presented here. Thus while incorporating a more general and complex ice-dynamics model could refine the details of these behaviors and allow fine-tuning of the other components of this hydromechanical model, we doubt that doing so would lead to significantly different conclusions about the mechanical behavior of till.

8. Conclusions

We have developed a hydromechanical model of the processes that govern basal motion of a soft-bedded alpine glacier. The simple three-column representation of the glacier bed has allowed us to model the dynamic response of a glacier to diurnal variations in subglacial drainage system pressure. Calculation of pore-water pressure, sediment deformation and sediment strength profiles, along with glacier sliding, permits us to model the responses of several commonly-used subglacial instruments. Comparison of modeled responses to field instrument records from Trapridge Glacier allows us to determine which flow law best describes typical instrument responses.

Of the four tested rheological models, modeled instrument responses for Coulomb-plastic till yield the best qualitative match to field instrument records from Trapridge Glacier. Synthetic tilt cell, slidometer and ploughmeter responses show good agreement in both magnitude and phase with field responses. Modeled responses for linear-viscous, nonlinear-viscous and nonlinear-Bingham tills fail to produce the observed phase relationship between system water pressure and ploughmeter force values, and till deformation rates for these laws are an order magnitude lower than those indicated by the field record.

For the modeled ranges of till thickness and basal pore-water pressure, Coulomb-plastic deformation occurs to depths of 0.05–0.76 m, with the "best fit" model indicating deformation to 0.35 m. These values show good agreement with the deformation depth of ~0.3 m estimated at Trapridge Glacier by *Blake* [1992]. Although at any one time deformation of Coulomb-plastic till can occur over a narrow band, migration of the actively-deforming zone with cyclic pressure variations leads to time-integrated deformation profiles that imply nearly linear-flow. This demonstrates that caution must be excercised when determining flow law parameters from long-term deformation profiles. Similarly, deformation in the uppermost portion of the till can lead to overestimation of the contribution of sliding to basal motion. This is especially true for Coulomb-plastic tills, which exhibit strong strain localization.

Unlike the other flow models, Coulomb-plastic till provides no additional resistance to glacier motion at elevated flow rates, making it more prone to unstable behavior. Modeling results suggest that, for Coulomb-plastic till, the connected region of the bed supports only ~20% of the basal driving stress during times of high drainage system pressure, with the unconnected region supporting a similar amount. These results are in agreement with estimates of basal stress supported by sediments beneath Antarctic ice streams by *Kamb* [1991] and *Tulaczyk et al.* [2000c] and highlight the importance of local pinning points in maintaining glacier stability. This raises the possibility that surge initiation could be a matter of overwhelming key pinning points rather than further softening of basal till.

Appendix A: Development of pore-water pressure evolution equation

.1.1. Till porosity and dilatancy

Development of the till porosity and dilatancy relations in this section closely follows *Clarke* [1987]. Subglacial sediment is considered a fully-saturated two-component mixture comprising water, having density $\rho_{\rm w}$ and occupying a volume fraction n, and solid particles having density $\rho_{\rm s}$ and occupying a volume fraction $1 - n$. We assume that the solid phase is incompressible and that water satisfies a simple equation of state

$$\rho_{\rm w}(p) = \rho_{\rm w}(p_0)\ \exp[\beta(p - p_0)] \tag{A1}$$

where β is the coefficient of compressibility, p the water pressure and p_0 a reference pressure value. The compressibility of the bulk water–solid mixture is a more subtle matter. By compressing the sediment and expelling water the bulk density is increased; by shearing it, assuming the mixture is dilatant, the porosity n is increased and bulk density decreases.

Although the water content of till is typically expressed in terms of porosity n, use of the void ratio e allows us to apply equations derived from geotechnical measurements of soil compressibility. Conversion between the two variables is a simple matter, as porosity and void ratio are related by

$$n = \frac{e}{1+e}. \tag{A2}$$

It follows that

$$\frac{\partial n}{\partial t} = \frac{1}{(1+e)^2}\frac{\partial e}{\partial t}. \tag{A3}$$

As noted earlier, subglacial water pressure is often expressed in terms of effective pressure $p' = p_I - p$, where p_I is the ice overburden pressure. Although water pressure is the measurable quantity, the effective pressure p' is of greater significance to soil mechanics because it represents the portion of the total pressure acting on the solid skeleton.

The compressibility α of a soil, which can be expressed as

$$\alpha = \frac{1}{1-n}\frac{\partial n}{\partial p}, \tag{A4}$$

is strongly dependent on the strain history of the soil. Compression of a soil that has never been subjected to loading (a "virgin soil") results in repositioning of grains in the solid matrix and thus a decrease in porosity or void ratio. Because the grains do not return to their original positions following subsequent unloading, the decrease in void ratio is largely irreversible. Following *Clarke* [1987], we express the void ratio of a virgin soil as

$$e = e_0 - \frac{1}{B_c}\ln\left[(p' + p_1')/p_0'\right], \tag{A5}$$

where B_c is a compression index and e_0 and p_0 are reference values of void ratio and effective pressure. The term p_1 prevents the void ratio from diverging as p' approaches zero, and can be determined by the void ratio e_1 at zero effective pressure:

$$p_1' = p_0' \exp\left[-B_c(e_1 - e_0)\right]. \tag{A6}$$

Equation (A5) defines the "normal consolidation line" (NCL; Fig. 13, long dashes), which relates the void ratio to effective stress for a virgin soil. States with lower void ratios for a given effective pressure are considered overconsolidated and are assumed to behave elastically so that further compaction under compression is recoverable. A "swelling index" B_s replaces B_c in the equation relating void ratio to effective pressure for overconsolidated states [*Clarke*, 1987]:

$$e(p', P') = e_0 - \left(\frac{1}{B_c} - \frac{1}{B_s}\right)\ln\left[(P' + p_1')/p_0'\right] - \frac{1}{B_s}\ln\left[(p' + p_1')/p_0'\right]. \tag{A7}$$

The term P' is the preconsolidation effective pressure, determined by

$$P' = \begin{cases} p' & \text{on NCL} \\ p'_0 \left[\exp\left(\frac{e - e_0}{\frac{1}{B_s} - \frac{1}{B_c}} \right) \right] \left(\frac{p' + p'_1}{p'_0} \right)^{\frac{1}{1-\frac{B_s}{B_c}}} - p'_1 & \text{left of NCL.} \end{cases} \quad \text{(A8)}$$

Because changes in effective pressure affect the void ratio of normally-consolidated and overconsolidated tills differently, the compressibility α depends on the consolidation state:

$$\alpha(p') = \begin{cases} \frac{1}{(1+e)B_c(p' + p'_1)} = \frac{1-n}{B_c(p' + p'_1)} & \text{on NCL} \\ \frac{1}{(1+e)B_s(p' + p'_1)} = \frac{1-n}{B_s(p' + p'_1)} & \text{left of NCL.} \end{cases} \quad \text{(A9)}$$

Many granular materials exhibit dilatancy, or a change in porosity upon shearing [*Reynolds*, 1885; *Andrade and Fox*, 1949]. Dilatant expansion can increase the porosity of an overconsolidated till, and thus basal deformation can influence the porosity and permeability structure of subglacial sediments. Given sufficient shearing, the void ratio of a soil will reach a critical state e_{CS} determined by the relation

$$e_{CS}(p') = e_0^\dagger - \frac{1}{B_c} \ln\left[(p' + p'_1)/p'_0\right] \quad \text{(A10)}$$

[*Clarke*, 1987] in which $e_0^\dagger$ is a reference void ratio value and B_c is the compression index used in (A5). Equation (A10) defines the "critical state line" (CSL; Fig. 13, short dashes), which is parallel to the NCL but yields slightly lower void ratio values for a given effective stress. States with lower void ratios for a given effective pressure CSL expand upon shearing, while states which lie between the CSL and NCL become more closely packed when sheared. We assume the rate of expansion or contraction to be proportional to both the strain rate in the soil and the difference between actual and critical-state void ratios:

$$\dot{e}_D(\dot{\varepsilon}) = -D_0 \dot{\varepsilon}(e - e_{CS}). \quad \text{(A11)}$$

Here $\dot{e}_D$ is the rate-of-change in void ratio due to dilatant reorganization and D_0 is a scaling factor. Differentiating (A5) and (A7) with respect to time and adding (A11) (and noting that $\partial p/\partial t = -\partial p'/\partial t$ for constant values of p_I) yields

$$\frac{\partial e}{\partial t} = \begin{cases} \frac{1}{B_s(p' + p'_1)} \dot{p} - D_0 \dot{\varepsilon}(e - e_{CS}) & \text{left of NCL} \\ \frac{1}{B_c(p' + p'_1)} \dot{p} - D_0 \dot{\varepsilon}(e - e_{CS}) & \text{on NCL.} \end{cases} \quad \text{(A12)}$$

It follows from (A3) that

$$\frac{\partial n}{\partial t} = \begin{cases} \frac{1}{(1+e)^2} \left[\frac{1}{B_s(p' + p'_1)} \dot{p} - D_0 \dot{\varepsilon}(e - e_{CS}) \right] & \text{left of NCL} \\ \frac{1}{(1+e)^2} \left[\frac{1}{B_c(p' + p'_1)} \dot{p} - D_0 \dot{\varepsilon}(e - e_{CS}) \right] & \text{on NCL,} \end{cases} \quad \text{(A13)}$$

Equation A13 allows us to relate the time-evolution of the porosity to the rate-of-change of pore-water pressure, the strain rate and the consolidation state.

.1.2. Hydraulic permeability

The porosity n of a soil gives an averaged measure of the packing of grains in the solid matrix. Because tight packing of the grains results in highly restricted water passageways, low-porosity soil will exhibit a low hydraulic permeability κ. Much effort has been made to determine the relationship between the porosity and permeability of a soil. Perhaps best known is the Kozeny-Carman relation [*Carman*, 1961]:

$$\kappa = \frac{n^3}{5(1-n)^2 S_0^2}. \quad \text{(A14)}$$

The term S_0 is the solid surface area per unit volume; the factor of 1/5 is that suggested by Carman. This relation has yielded reasonable estimates of porosity for clean sands [*Bourbié et al.*, 1987], but its applicability to tills remains unclear.

.1.3. Lagrangian representation of water transport

The usual approach to modeling water flux through a porous medium is to cast the problem in terms of the Eulerian, or spatial, description. This approach is appropriate for elastic aquifers that experience only small strains, in which case the distinction between unstrained and strained states can be neglected. For materials that can undergo a large volume change, such as dilatant till, this simplification is inappropriate, and thus we use a Lagrangian representation of till. With this approach, spatial and temporal variability are expressed in terms of the initial configuration so that $p(X_k, t)$ describes the spatial and temporal distribution of pressure. To be mathematically rigorous, it is customary to emphasize the fact that $p(x_k, t)$ and $p(X_k, t)$ are different mathematical functions by introducing two different pressure functions that describe the same physical property. In the following discussion, Eulerian functions are denoted by a "tilde" (e.g., in the Eulerian scheme $n = \tilde{n}(x_k, t)$, and in the Lagrangian $n = n(X_k, t)$).

We assume that properties of the bed are uniform for some distance both up- and down-glacier from the location of the till column. Thus as deformation within the column moves sediments down-glacier and therefore away from the one-dimensional column, those sediments are immediately replaced by sediments with identical properties. With this assumption, we can express p and other physical properties as functions their height (Z, t) within the till column, where Z is the initial position of a particle in the column. The height of such a particle changes with time in response to dilatation and compression, and thus a solid particle having initial position Z will at some later time be located at z, where

$$z(Z, t) = \int_0^Z \left(\frac{1 - n(Z', 0)}{1 - n(Z', t)} \right) dZ'. \quad \text{(A15)}$$

From (A15) the Jacobean of the transformation from Z to z is

$$J(t) = \frac{\partial z}{\partial Z} = \frac{1 - n(Z, 0)}{1 - n(Z, t)}. \quad \text{(A16)}$$

If we make the usual assumption that solids are incompressible, the solid mass m_s per unit area of bed is given in the material description by

$$m_s = \rho_s \int_0^{h(0)} \big(1 - n(Z,t)\big) J(t)\, dZ. \qquad \text{(A17)}$$

The condition for conservation of solid mass is $dm_s/dt = 0$, which leads to the local-form expression

$$\frac{\partial}{\partial t}\Big[\big(1 - n(Z,t)\big) J(t)\Big] \equiv 0. \qquad \text{(A18)}$$

Equation (A18) is an identity: solid mass is automatically conserved in the Z–t coordinate system. Thus in the Lagrangian representation the solid mass balance condition is automatically satisfied and the water balance condition is the one that leads to field equations. In the Lagrangian representation the water mass (per unit area of bed) is

$$m_w = \int_0^{h(0)} \rho_w(Z,t) n(Z,t) J(t)\, dZ. \qquad \text{(A19)}$$

Within the region $0 \le Z \le h(0)$ water mass varies because of dilatation or compression of the till layer. This change in water mass is a consequence of the fact that water moves independently of the solid matrix. As the pore volume in a given region increases or decreases in response to dilatation or contraction of the solid matrix, water flows in or out of the region to accommodate the changes in pore volume. The change in total water mass for column of till is given by the difference in water mass flux $\rho_w q_w$ between the upper and lower boundaries of the column $Z = 0$ and $Z = h$; here q_w denotes the volume flux of water. Thus

$$\begin{aligned} \frac{dm_w}{dt} &= -\rho_w(h,t) q_w(h,t) + \rho_w(0,t) q_w(0,t) \qquad \text{(A20)} \\ &= -\int_0^{h(0)} \frac{\partial}{\partial Z}\Big(\rho_w(Z,t) q_w(Z,t)\Big)\, dZ. \end{aligned}$$

Equations (A19) and (A20) lead to the local form expression of water balance

$$\frac{\partial}{\partial t}\{\rho_w(Z,t) n(Z,t) J(t)\} = -\frac{\partial}{\partial Z}\{\rho_w(Z,t) q_w(Z,t)\} \qquad \text{(A21)}$$

From (A21) it is clear that changes in pore volume due to dilatation or compression result in non-zero vertical water flux values. From (A1) we get

$$\frac{\partial \rho_w}{\partial t} = \beta \rho_w \frac{\partial p}{\partial t} \qquad \text{(A22)}$$

$$\frac{\partial \rho_w}{\partial Z} = \beta \rho_w \frac{\partial p}{\partial Z}. \qquad \text{(A23)}$$

We assume that transport of water in the cross- or down-glacier directions is negligible compared to vertical flow driven by diurnal pressure variations. Water flux q_w in the column is assumed to be governed by Darcy's law, which in the material description is given by

$$q_w(Z,t) = -\frac{\kappa(Z,t)}{\mu_w}\left[\frac{1}{J(t)}\frac{\partial p(Z,t)}{\partial Z} + \rho_w g\right]. \qquad \text{(A24)}$$

Expanding the left-hand side of (A21) and using relations (A4), (A16) and (A22) leads to

$$\frac{\partial}{\partial t}\left(\rho_w(Z,t) n(Z,t) J(t)\right) = \rho_w(Z,t)\left[\alpha(p) + n(Z,t)\beta\right] J(t) \frac{\partial}{\partial t} p(Z,t). \qquad \text{(A25)}$$

Expanding the right-hand side of (A21) and incorporating (A23) gives

$$-\frac{\partial}{\partial Z}\Big(\rho_w(Z,t) q_w(Z,t)\Big) = -\rho_w(Z,t)\left[q_w(Z,t)\beta \frac{\partial}{\partial Z} p(Z,t) + \frac{\partial}{\partial Z} q_w(Z,t)\right]. \qquad \text{(A26)}$$

Equating (A25) and (A26), substituting the Lagrangian expression of Darcy's law (A24), and rearranging yields the pressure evolution equation (7).

Appendix B: Model sensitivity

Modeled responses discussed in the preceding sections were calculated using parameter values listed in Tables 1, 2 and 3. In this section we summarize the effects of varying the till flow law parameters and several key tuning parameters.

.2.1. Sensitivity to till flow law parameters

2.1.1. Linear-viscous till

Estimates of effective till viscosity (Eq. 1) range between 2×10^8 and 5×10^{11} Pa s [*Paterson*, 1994, Table 8.2]; values estimated from Trapridge Glacier studies range from 3×10^9 [*Fischer and Clarke*, 1994] to 1.5×10^{11} Pa s [*Blake*, 1992]. Results presented above assume a linear viscosity of $\eta_0 = 2.0 \times 10^{10}$ Pa s. Decreasing this value by an order of magnitude changes modeled responses considerably. At this viscosity value, soft-bedded regions support less than 5% of the driving stress, resulting in increased ice velocities and deformation rates. The relative contribution of glacier sliding to the total basal motion decreases, and maximum ploughmeter force values, at 0.12–0.25 kN, are an order of magnitude lower. Increasing the viscosity to $\eta_0 = 2.0 \times 10^{11}$ Pa s produces opposite effects, leading to lower deformation rates and maximum ploughmeter force values measuring 10.2–10.6 kN, ~5 times greater than those calculated for the nominal viscosity value. Modeled ice velocities, sliding rates and ploughmeter force values at an effective viscosity of $\eta_0 = 2.0 \times 10^{10}$ Pa s most closely match Trapridge Glacier observations.

2.1.2. Nonlinear-viscous till

The stiffness of nonlinear-viscous till is determined by the value of B_1 in (2). The value used for modeling the responses presented above, at $1.0 \times 10^3\,(\mathrm{kPa})^{0.47}\mathrm{a}^{-1}$, is ~30 times that suggested by *Boulton and Hindmarsh* [1987]. Increasing this value by an order of magnitude produces changes similar to those resulting from decreasing the effective viscosity of a linear-viscous till, yielding higher flow velocities, increased deformation rates and lower ploughmeter force values. Ploughmeter responses are ~15% of those calculated for the nominal value, with peak force values of 0.5–2.1 kN. Soft-bedded regions support only 7–22% of the driving stress at this value of B_1. Decreasing the value of B_1 by an order of magnitude produces reduced ice velocities and deformation rates and peak ploughmeter responses ranging between 29.0 and 59.6 kN, ~5–7 times greater than those for the nominal value of B_1.

2.1.3. Nonlinear-Bingham till

Modeled deformation rates for nonlinear-Bingham till are 4–10% of those indicated by BT1 using the value B_2 =

121.0 (kPa)$^{0.625}$a^{-1} in (3) suggested by Boulton and Hindmarsh. Because the strain rate is related to the reduced shear stress $\tau - \sigma_Y$, this flow law is stiff for low values of B_2. Increasing the value of B_2 to soften the till and encourage greater deformation results in what is essentially Coulomb-plastic behavior: when the shear stress reaches the yield strength σ_Y of the material, deformation proceeds at a rapid rate, preventing the applied stress from exceeding the yield strength. Increasing the value of B_2 by an order of magnitude results in instrument responses similar to those for Coulomb-plastic till of the same yield strength σ_Y.

2.1.4. Coulomb-plastic till

Because of the rate-independent nature of Coulomb-plastic till, modeled ice velocities show little dependence on the yield strength σ_Y in (4). Increasing the value of ϕ in (4) limits basal deformation in the connected region during times of low system pressure but does not strongly affect sliding or deformation rates at high system pressures. For values of ϕ greater than ~30°, (assuming $c_0 = 0$), the yield strength is sufficiently high to limit deformation to the uppermost 0.03 m of the till layer and thus preclude tilt cell rotation. High basal water pressures in the unconnected region allow deformation for all reasonable yield strength values.

.2.2. Sensitivity to model tuning parameters

2.2.1. Pore-water pressure at till base p_B

The value prescribed to the pore-water pressure p_B at the base of the till layer determines both the pressure gradient within the layer and, for nonlinear-viscous, Bingham and plastic tills, stiffness of the till at depth. Because the effective pressure does not appear in the flow relation (1) for linear-viscous till, no changes in modeled responses are noted with variations in basal pressure. For the pressure-dependent flow laws, lower basal pressures yield steeper pressure gradients and slightly lower deformation rates, with slight increases noted in ploughmeter force values. Soft-bedded regions for nonlinear-viscous and Bingham tills support marginally greater portions of the basal shear stress due to till stiffening at depth. Higher basal pressures produce opposite results, yielding higher deformation rates, lower ploughmeter force values and lower basal stresses in soft-bedded regions for nonlinear-viscous and Bingham tills. These tills also exhibit increased total ice velocities with increased basal pressure. For Coulomb-plastic till, variations in basal pressure do not noticeably affect the total ice velocity.

The depth to which diurnal pressure fluctuations penetrate into the till layer is largely determined by the steepness of the pressure gradient. Low basal pressures (and steep pressure gradients) result in shallow penetration depths, with deeper penetration resulting from higher basal pressures. Because till strength depends on pore-water pressure, the basal pressure p_B plays a large role in determining the depth to which deformation occurs in the till layer. For nonlinear-viscous tills, the depth above which 50% of the total deformation occurs is 0.31 m at $p_B = 30.0$ m, the default value. This depth increases to 0.42 m at $p_B = 45.0$ m and decreases to 0.25 m at $p_B = 15.0$ m. This effect is more pronounced for Bingham and Coulomb-plastic tills. For nonlinear-Bingham till, deformation occurs to a depth of 0.45 m for $p_B = 30.0$; values for p_B of 15.0 m and 45.0 m result in maximum deformation depths of 0.45 m and 0.94 m, respectively. Deformation of Coulomb-plastic till occurs to depths of 0.76 m, 0.35 m and 0.16 m at basal pressures of 45.0, 30.0 and 15.0 m.

2.2.2. Till thickness h

For linear-viscous and nonlinear-viscous tills, depth-integrated deformation rates scale roughly with layer thickness. Total basal deformation rates for linear-viscous till average 0.061 m d^{-1} for till layer thickness of $h = 1.0$ m. Halving the layer thickness to 0.50 m reduces the total deformation rate to 0.037 m d^{-1}, while doubling the thickness to 2.0 m increases the total deformation rate to 0.091 m d^{-1}. The relationship between layer thickness and deformation rate is not 1:1 because any adjustment in flow velocity results in redistribution of basal shear stresses. Nonlinear-viscous till exhibits average total basal deformation rates of 0.020 m d^{-1} for a layer thickness of 0.50 m, 0.036 m d^{-1} at 1.00 m and 0.057 m d^{-1} at 2.00 m. Because deformation rates and till characteristics in the upper portion of the till layer are similar for these layer thicknesses, modeled instrument responses show only minor variations with till-layer thickness.

Changes in till thickness affect Bingham and Coulomb-plastic tills mainly through their influence on the pore-water pressure gradient: for a given value of basal pore-water pressure p_B, greater till thicknesses result in lower pressure gradients and thus deeper deformation. For nonlinear-Bingham tills, deformation occurs to depths of 0.12 m in a till layer of thickness 0.50 m, to 0.45 m in a till of thickness 1.00 m and 0.75 m in a 2.00 m-thick till. For Coulomb-plastic till, layer thicknesses of 0.50, 1.00 and 2.00 m yield deformation to depths of 0.07, 0.35 and 0.60 m. The steep pressure gradient resulting from decreasing the till layer thickness to 0.50 m till yields rapid stiffening of the till with depth for both till models, resulting in higher ploughmeter force values and lower deformation rates at depth; till stiffness for both flow relations is sufficient to preclude any deformation at the greatest modeled tilt cell installation depth. The lower pressure gradient in the the 2.00 m layer thickness results lower ploughmeter force values and increased deformation rates at depth. Although the total ice velocity for Bingham till scales roughly with till thickness, Coulomb-plastic till shows no dependence of ice velocity on layer thickness.

2.2.3. Basal lengthscales L_H and L_S

The ice flow rate is determined to a large extent by the value chosen for L_H, which represents the average length scale of variation between soft- and hard-bedded regions. The nominal value $L_H = 1500$ m yields average modeled ice velocities ranging between 0.04 m d^{-1} for Bingham till to 0.08 m d^{-1} for linear-viscous till. Increasing the length scale to $L_H = 3000.0$ m results in a moderate increase in ice velocities, to 0.06–0.16 m d^{-1}; modeled deformation rates, sliding rates and ploughmeter force values are ~150–200% those calculated for the nominal value of L_H. Reducing the length scale to $L_H = 750.0$ m yields ice velocities of 0.03–0.05 m d^{-1} and modeled instrument responses ~40–70% of those for the nominal value of L_H. At the longer length scale, a slight increase is noted in the shear stress supported by soft-bedded regions, resulting from increased viscous drag at the higher deformation rates. At the shorter length scale, decreased ice-deformation rates result in lower shear stresses in these regions. Because the strength of Coulomb-plastic till is independent of deformation rate, the shear stress supported by soft-bedded regions does not vary with L_H for this flow law.

The value chosen for the length scale of variation between connected and unconnected regions L_S has little effect on modeled instrument responses or shear stress distribution. This parameter determines the velocity difference between soft-bedded regions. The nominal value $L_S = 150$ m yields velocity contrasts at minimum system pressure measuring <1–22% of the total ice velocity. Increasing L_S to 300.0 m results in velocity differences of <1–48%; decreasing L_S to 75.0 m yields differences ranging between <1% and 10%.

2.2.4. Areal fractions α_i

Increasing the fraction α_1 of hydraulically-connected regions while decreasing the fraction α_2 of unconnected regions effectively increases the proportion of the bed that drives glacier motion during times of high system pressure while decreasing the fraction driving motion at low system pressures. As a result, larger values of α_1 yield both higher peak ice velocities and decreased flow rates at times of low system pressure. In the case of Bingham and Coulomb-plastic tills, increasing α_1 such that $\alpha_1/\alpha_2 > \sim 3.5$ results in driving stresses below the till yield strength σ_Y during times of low system pressure. Because the field record for BT1 (Fig. 2b) indicates that deformation does occur at times of low system pressures, this suggests an upper limit to the ratio α_1/α_2 of connected to unconnected regions for Bingham and Coulomb-plastic tills.

Varying the areal fraction α_3 of hard-bedded regions has no effect on either glacier motion or instrument response for any of the four modeled flow laws. When α_3 is varied while the ratio of connected to unconnected regions is held at $\alpha_1/\alpha_2 = 1/3$, only the basal stress τ_3 supported by the hard-bedded region varies with changes in α_3 – no change is noted in the shear stress supported by either of the soft-bedded regions. This result is not surprising, as the flow rate of the glacier is controlled by (1) the ice properties, (2) the variational lengthscale L_H and (3) the sediment properties. Doubling the nominal value of α_3 from 0.20 to 0.40 results in diurnally-varying values for τ_3 of 2.1–2.2 τ_0 for linear-viscous till and 2.0–2.1 τ_0 for nonlinear-viscous till. Nonlinear-Bingham and Coulomb-plastic tills give values for τ_3 ranging 1.8–2.1 τ_0 and 2.0–2.3 τ_0, respectively. Decreasing the areal fraction of hard-bedded regions to $\alpha_3 = 0.10$ greatly increases the stress supported by hard-bedded regions: for a Coulomb-plastic till at peak drainage system pressure, the hard-bedded regions support a stress 8.8 times the nominal driving stress. For the other flow laws, peak stresses range between 7.3 τ_0 and 8.0 τ_0. These tests show that the magnitude of shear stress supported by the hard-bedded region depends strongly on the value of α_3 chosen, while the stresses supported by soft-bedded regions are independent of this value. If a glacier is predominantly soft-bedded, as Trapridge Glacier appears to be, hard-bedded regions could support very high stress values. Failure of the these pinning points – or of the ice coupling them to the soft-bedded regions – could result in unstable behavior.

2.2.5. Flotation-pressure sliding coefficient C_{SL}

The sliding coefficient C_{SL} in (12) determines the amount of sliding for given shear stress and subglacial water pressure values. While it would thus be reasonable to assume that the value of C_{SL} would have a large influence on the ice velocity V, modeling scenarios show that variations in C_{SL} yield only minor changes in V. Doubling the value of C_{SL} at $p = p_I$ to 40% results in only a slight (<4%) increases in ice velocity at peak system pressures for all flow laws. Similarly, decreasing the flotation-pressure value to $C_{SL} = 10\%$ yields ice velocities only 1–3% lower. These minor responses are due to the fact that the ice velocity is ultimately limited by the mechanical properties of the ice: given strong pinning points (here provided by the hard-bedded region) and neglecting fracturing, the ice will only flow as rapidly as deformation between columns will allow.

Although variations in the degree of ice–bed coupling result in only minor changes to the total ice velocity, they do have a major effect on how that velocity is divided between sliding and sediment deformation: With viscous coupling providing 20% of the total ice–bed coupling at flotation pressure, sliding accounts for 55% of the total ice velocity for linear-viscous till at the maximum drainage system pressure. At 10% viscous coupling, sliding comprises 37% of basal motion; at 40%, sliding accounts for 73%. This effect is largest for Coulomb-plastic till, with sliding contributing 23, 47 and 99% of the total basal motion at flotation-pressure C_{SL} values of 10, 20 and 40%. It should be noted, however, that because modeled deformation of Coulomb-plastic till occurs in the top 0.01–0.03 m of the till layer at high system pressures, there is little practical difference between deformation and sliding.

2.2.6. Unconnected-region sliding

In all of the model runs discussed above, it was assumed that no sliding occurs in the unconnected region. Even with this assumption, only a small portion of the driving stress is supported by the unconnected region because all four till flow laws yield relatively weak tills. Allowing sliding in the unconnected region results in an additional reduction of the shear stress supported there. Table 6 shows the minimum and maximum shear stresses supported by each region for modeling scenarios in which basal sliding is allowed in the unconnected region. In comparison to the non-sliding case (Table 5), reductions in the shear stress supported by the unconnected region is reduced by ∼40% for linear-viscous, nonlinear-viscous and nonlinear-Bingham tills, with the balance of the shear stress redistributed onto the hard-bedded region and, to a lesser extent, connected regions. No change in the stress supported is indicated for Coulomb-plastic till. It can also be seen in Table 6 that sliding in the unconnected region further increases the role of hard-bedded regions in maintaining glacier stability.

With sliding allowed, the unconnected region provides modeled peak ice velocities are 30%, 31% and 45% greater than the non-sliding case for linear-viscous, nonlinear-viscous and nonlinear-Bingham tills, respectively. No significant change in peak ice velocity is seen for Coulomb-plastic till. Minimum ice velocities also show increased values; of the flow laws modeled, nonlinear-Bingham till shows the largest increase, with a minimum ice velocity 166% of that modeled in the non-sliding case.

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Figure 1. Study area. (a) Location map, southwestern Yukon Territory, Canada. (b) Trapridge Glacier, showing instrumented region of the glacier. (c) Study site.

Figure 2. Field instrument records during 20–25 July (days 202–207) 1996. (a) Records for P1 (solid line) and PZ1 (dashed line). (b) Strain rate in till calculated from the tilt record for BT1 (solid line). (c) Sliding rate determined from displacement record for SL1. (d) Force record for PL1 (solid line). The pressure record for P1 is shown as the dashed lines in (b–d).

Figure 3. Coordinate system for soft-bedded regions. Hard-bedded regions are similar, but with no till layer.

Figure 4. Stress-strain rate relations for modeled till flow laws. (a) Linear-viscous till. (b) Nonlinear-viscous till, with $a = 1.33$ and $b = 1.80$. (c) Nonlinear-Bingham till, with $a = 0.625$ and $b = 1.25$. Shaded region represents transition zone between elastic ($\tau < \sigma_Y$) and viscous ($\tau > \sigma_{min}$) behaviors. (d) Coulomb-plastic till.

Figure 5. Plan view of glacier bed showing till column arrangement used to determine stress transfer for column 2. Glacier flow is oriented towards the top of the page. Columns 1 and 2 are situated in connected (light grey) and unconnected (mid-tone grey) soft-bedded regions; column 3 is in a hard-bedded region (dark grey). Soft-bedded columns 1 and 2 are separated by distance L_S; hard-bedded column 3 is distance L_H from column 2.

Figure 6. Modeled pore-water pressure and deformation profiles. (a–d) Modeled pore-water pressure profiles in the connected region. Pressure value in shaded region above $Z = 1.0\,m$ represents the flotation pressure. (a) Profiles at $t = 0$, corresponding to peak system pressure (solid line), and $t = 0.125$ d (dashed line). (b) Profiles at $t = 0.250$ d (solid line) and $t = 0.375$ d (dashed line). (c) Profiles at $t = 0.500$ d (minimum system pressure; solid line) and $t = 0.375$ d (dashed line). (d) Profiles at $t = 0.750$ d (solid line) and $t = 0.875$ d (dashed line). (e) Modeled pressure profile in unconnected region. for times of maximum (solid line) and minimum (dashed line) system pressure. (f–y) Modeled till deformation profiles. Total ice velocity is shown in shaded region above $Z = 1.0$ m. (f–i) Connected-region deformation profiles for linear-viscous till at times corresponding to pressure profiles in (a–d). (j) Deformation profiles in unconnected region at times of maximum (solid line) and minimum (dashed line) system pressure modeled for linear-viscous till. (k–o) Deformation profiles for nonlinear-viscous till; times and connection status correspond to (a–e). (p–t) Deformation profiles for nonlinear-Bingham till. (u–y) Deformation profiles for Coulomb-plastic till.

Figure 7. Total till displacement profiles during one model day for connected region (a–d) and unconnected region (e–h). Profiles (a,e) correspond to linear-viscous till, (b,f) to nonlinear-viscous till, (c,g) to nonlinear-Bingham till and (d,h) to Coulomb-plastic till.

Figure 8. Modeled five-day basal shear stress for (a) linear-viscous, (b) nonlinear-viscous, (c) nonlinear-Bingham, and (d) Coulomb-plastic tills in connected (solid lines), unconnected (short dashes) and hard-bedded (long dashes) regions. Modeled drainage system pressure (thick grey line) shown for comparison.

Figure 9. Comparison of field and modeled pore-water pressure records. (a) Records for P1 (solid line) and PZ1 (dashed line) during 20-25 July (days 202–207) 1996. Transducer PZ1 was installed at an estimated depth of 0.15 m. (b) Five-day pore-water pressures modeled for linear-viscous till at depths of 0.15 (short dashes), 0.25 (long dashes) and 0.35 m (short-long dashes). The solid line represents the drainage system pressure. Pore-water pressures for other till flow laws are similar.

Figure 10. Comparison of till strain rates calculated from field and modeled tilt cell records. (a) Strain rate record for BT1 (black line) during 20-25 July (days 202–207) 1996. The pressure record for P1 (grey line) shown for comparison. Tilt cell BT1 was installed at an estimated depth of 0.15 m. (b–e) Modeled five-day strain rates for installation depths of 0.15 (solid line), 0.25 (short dashes) and 0.35 m (long dashes). Modeled drainage system pressure (thick grey line) shown for comparison. (b–e): Modeled strain rates for (b) linear-viscous, (c) nonlinear-viscous, (d) nonlinear-Bingham, and (e) Coulomb-plastic tills.

Figure 11. Comparison of indicated sliding rates calculated from field and modeled slidometer records. (a) Indicated sliding rates for SL1 (black line) during 20-25 July (days 202–207) 1996. Pressures reported by P1 (grey line) shown for comparison. The anchor for slidometer SL1 was installed at an estimated depth of 0.12 m. (b–e) Modeled five-day sliding rates for installation depths of 0.10 (short dashes), 0.20 (long dashes) and 0.30 m (short-long dashes). The true sliding rate is shown as the solid line, with the modeled drainage system pressure (thick grey line) included for comparison. (b–e): Indicated strain rates for linear-viscous (b), nonlinear-viscous (c), nonlinear-Bingham (d), and Coulomb-plastic (e) tills.

Figure 12. Comparison of field and modeled ploughmeter records. (a) Force record for PL1 (black line) during 20-25 July (days 202–207) 1996. The pressure record for P1 (grey line) shown for comparison. Ploughmeter PL1 was installed at an estimated depth of 0.14 m. (b–e) Modeled five-day ploughmeter records for installation depths of 0.10 (solid line), 0.20 (short dashes) and 0.30 m (long dashes). Modeled drainage system pressure (thick grey line) shown for comparison. (b–e): Modeled force records for linear-viscous (b), nonlinear-viscous (c), nonlinear-Bingham (d), and Coulomb-plastic (e) tills.

Figure 13. Compressibility characteristics of modeled till, adapted from Clarke (1987, Fig. 1). The normal consolidation line (NCL, long dashes) relates porosity to effective pressure for virgin, or previously uncompressed, till. The solid lines intersecting the NCL at A and B ("swelling lines") demonstrate compressibility characteristics of overconsolidated tills. If a normally consolidated till at point A is subjected to increasing effective pressure, porosity decreases until point B is reached. The effective pressure corresponding to point B is called the preconsolidation pressure. If the effective pressure is subsequently reduced, the former compression state is not recoverable, and the state path follows a swelling line to point C. If effective pressure is then increased, the state path again traverses the swelling line to point B and then follows the normal consolidation line. The critical state line (CSL, short dashes) represents the porosity of a fully dilated till. Shearing drives the consolidation state vertically towards the CSL at a rate proportional to both the rate of shearing and the distance from the CSL. States lying above the CSL compact upon deformation; porosity of states lying below the CSL increase with deformation.

Table 1. Physical constants used in the Trapridge Glacier model.

Constant	Value	Description
g	$9.81\,\mathrm{m\,s^{-2}}$	Gravitational acceleration
ρ_w	$1000.0\,\mathrm{kg\,m^{-3}}$	Density of water
β	$5.10 \times 10^{-10}\,\mathrm{Pa^{-1}}$	Compressibility of water
μ_w	$1.787 \times 10^{-3}\,\mathrm{Pa\,s}$	Viscosity of water
ρ_I	$900\,\mathrm{kg\,m^{-3}}$	Density of ice
G_I	3.5×10^9	Rigidity modulus of ice[a]
A_0	$1.6 \times 10^{-15}\,(\mathrm{kPa})^{-3}\mathrm{s}^{-1}$	Glen flow parameter[b]
N	3.00	Glen flow exponent[b]
ρ_s	$2800\,\mathrm{kg\,m^{-3}}$	Density of sediment[c]
S_0	$5.00 \times 10^6\,\mathrm{m^{-1}}$	Surface-to-volume ratio of sediments[c]
e_0	0.266	Reference value void ratio for NCL[c]
p'_0	500.0 kPa	Reference value of effective pressure[c]
e_1	0.389	Zero effective pressure void ratio[c]
$e_0^\dagger$	0.250	Reference value of void ratio for CSL[c]
B_c	32.9	Compression index[d]
B_s	164.0	Swelling index[d]
D_0	0.20	Dilatant response scaling factor[c]

[a] Average value for −5°C ice from *Ewing et al.* [1934], *Northwood* [1947], *Jona and Scherrer* [1952] and *Gold* [1958].

[b] From *Paterson* [1994, p. 97].

[c] See text.

[d] From *Clarke* [1987], based on studies of Glasgow till [*McKinlay et al.*, 1978, Table 3].

Table 2. Assumed till flow law parameters for the Trapridge Glacier model.

Parameter	Value	Description
Linear-viscous till:[a]		
η_0	2.0×10^{10} Pa s	Till viscosity
Nonlinear-viscous till:[b]		
B_1	1.0×10^3 (kPa)$^{0.47}$a^{-1}	Flow parameter
a	1.33	Shear stress exponent
b	1.80	Effective pressure exponent
Nonlinear-Bingham till:[c]		
B_2	121.0 (kPa)$^{0.625}$a^{-1}	Flow parameter
a	0.625	Shear stress exponent
b	1.25	Effective pressure exponent
Mohr-Coulomb failure criterion parameters:[d]		
c_0	0.00	Residual cohesion constant
ϕ	18.6°	Residual friction angle
Coulomb-plastic till parameterization:[e]		
$\dot{\varepsilon}_0$	1.0×10^{-4} s^{-1}	Reference strain rate
$\Delta\tau$	5.0 kPa	Failure range

[a] From Blake (1992), Fischer and Clarke (1994).

[b] Boulton and Hindmarsh (1987); value for B_1 is ~30 times greater than that suggested (see text).

[c] Iverson et al. (1998).

[d] Boulton and Hindmarsh (1987).

[e] See text.

Table 3. Model parameters for response tests.

Parameter	Value	Description
Model geometry:		
θ	7.0°	Ice surface slope
H_I	70.0 m	Thickness of ice
h	1.0 m	Thickness of till layer
Areal fractions:		
α_1	0.20	Areal fraction of connected region
α_2	0.60	Areal fraction of unconnected region
α_3	0.20	Areal fraction of hard-bedded region
Length scales of basal variation:		
L_H	1500.0 m	Hard-bedded and soft-bedded variational scale
L_S	150.0 m	Soft-bedded variational scale
Glacier sliding parameters:		
p_{SL}	698.5 kPa	Reference pressure for glacier sliding
Δp_{SL}	629.1 kPa	Pressure range for glacier sliding
h_{SL}	0.01 m	Thickness of lubricating layer
μ_{SL}	2.0×10^7 Pa s	Viscosity of lubricating layer
Connected region water pressure:		
p_{T1}	490.0 kPa	Mean water pressure at $Z = h$
Δp_{T1}	98.0 kPa	Amplitude of diurnal pressure variation
p_{B1}	294.0	Water pressure at $Z = 0$
Unconnected region water pressure:		
p_{T2}	588.0 kPa	Mean water pressure at $Z = h$
p_{B2}	294.0	Water pressure at $Z = 0$

Table 4. Modeled deformation, sliding and total ice displacement rates.

Flow Law	Deformation Rate, Connected Region	Sliding Rate, Connected Region	Total Displacement Rate, Connected Region	Deformation Rate, Unconn. Region
Linear-viscous:				
Max. System Pressure:	$0.038\,\mathrm{m\,d^{-1}}$	$0.046\,\mathrm{m\,d^{-1}}$	$0.084\,\mathrm{m\,d^{-1}}$	$0.084\,\mathrm{m\,d^{-1}}$
Min. System Pressure:	0.076	0.002	0.078	0.078
Nonlinear-viscous:				
Max. System Pressure:	0.021	0.058	0.079	0.079
Min. System Pressure:	0.048	0.005	0.053	0.055
Nonlinear-Bingham:				
Max. System Pressure:	0.002	0.063	0.065	0.064
Min. System Pressure:	0.013	0.009	0.022	0.027
Coulomb-plastic:				
Max. System Pressure:	0.061	0.054	0.115	0.114
Min. System Pressure:	0.036	0.008	0.044	0.049

Table 5. Modeled basal shear stresses.

Flow Law	Minimum Shear Stress	Maximum Shear Stress
Linear-viscous:		
Connected region	$0.12\,\tau_0$	$0.23\,\tau_0$
Unconnected region	0.24	0.26
Hard-bedded region	4.05	4.11
Nonlinear-viscous:		
Connected region	0.14	0.63
Unconnected region	0.21	0.28
Hard-bedded region	3.73	4.03
Nonlinear-Bingham:		
Connected region	0.16	1.02
Unconnected region	0.24	0.36
Hard-bedded region	3.22	3.81
Coulomb-plastic:		
Connected region	0.13	0.94
Unconnected region	0.13	0.13
Hard-bedded region	3.67	4.45

Table 6. Modeled basal shear stresses with sliding allowed in the unconnected region.

Flow Law	Minimum Shear Stress	Maximum Shear Stress
Linear-viscous:		
Connected region	0.15 τ_0	0.30 τ_0
Unconnected region	0.14	0.15
Hard-bedded region	4.29	4.40
Nonlinear-viscous:		
Connected region	0.19	0.73
Unconnected region	0.11	0.16
Hard-bedded region	3.93	4.33
Nonlinear-Bingham:		
Connected region	0.21	1.09
Unconnected region	0.11	0.20
Hard-bedded region	3.56	4.19
Coulomb-plastic:		
Connected region	0.14	0.94
Unconnected region	0.12	0.13
Hard-bedded region	3.71	4.47