Geomorphic Transport Laws for Predicting Landscape Form and Dynamics

William E. Dietrich, Dino G. Bellugi, Leonard S. Sklar, Jonathan D. Stock

Department of Earth and Planetary Science, University of California, Berkeley, California

Arjun M. Heimsath

Department of Earth Sciences, Dartmouth University, Hanover, New Hampshire

Joshua J. Roering

Department of Geological Sciences, University of Oregon, Eugene, Oregon

A geomorphic transport law is a mathematical statement derived from a physical principle or mechanism, which expresses the mass flux or erosion caused by one or more processes in a manner that: 1) can be parameterized from field measurements, 2) can be tested in physical models, and 3) can be applied over geomorphically significant spatial and temporal scales. Such laws are a compromise between physics-based theory that requires extensive information about materials and their interactions, which may be hard to quantify across real landscapes, and rules-based approaches, which cannot be tested directly but only can be used in models to see if the model outcomes match some expected or observed state. We propose that landscape evolution modeling can be broadly categorized into detailed, apparent, statistical and essential realism models and it is the latter, concerned with explaining mechanistically the essential morphodynamic features of a landscape, in which geomorphic transport laws are most effectively applied. A limited number of studies have provided verification and parameterization of geomorphic transport laws for: linear slope-dependent transport, non-linear transport due to dilational disturbance of soil, soil production from bedrock, and river incision into bedrock. Field parameterized geomorphic transport laws, however, are lacking for many processes including landslides, debris flows, surface wash, and glacial scour. We propose the use of high- resolution topography, as initial conditions, in landscape evolution models and explore the applicability of locally parameterized geomorphic transport laws in explaining hillslope morphology in the Oregon Coast Range. This modeling reveals unexpected morphodynamics, suggesting that the use of real landscapes with geomorphic transport laws may provide new insights about the linkages between process and form.

Prediction in Geomorphology
Geophysical Monograph 135

10.1029/135GM09

1. INTRODUCTION

A debate is underway about what modeling approaches are necessary or appropriate for explaining and predicting

landscape form and evolution. This debate is partly focused on what are the compelling questions [e.g. *Rodriguez-Iturbe* and *Rinaldo*, 1997]. It is also about what degree of physical approximation is acceptable when searching for mechanistic explanation (as will be discussed at length in this book). Here we argue for the value of developing and applying what can be called geomorphic transport laws.

The use of transport laws in the conservation of mass equation to explore controls on landscape form and dynamics was introduced by *Culling* [1960]. Subsequently, both *Kirkby* [1971] and *Smith* and *Bretherton* [1972] proposed generalized transport laws that distinguished hillslope and channel processes primarily by their drainage area dependency. These authors recognized that specific transport laws will produce, under specific boundary conditions, a 'characteristic form' [*Kirkby*, 1971] such that the shapes of hillslopes or channels are the pure expression of the transport law (a view first argued by *Gilbert* [1877, 1909] and *Davis* [1892]). The basic approach of using transport laws in conservation equations pioneered by these authors is now commonly applied in numerical models developed to tackle a wide range of problems [e.g. *Ahnert*, 1988; *Willgoose et al.*, 1991a, b, c; *Tucker* and *Slingerland*, 1994, 1997; *Anderson*, 1994; *Howard*, 1994; *Kooi* and *Beaumont*, 1996; *van der Beek* and *Braun*, 1999].

Underlying all these approaches is the assumption that these transport laws are sufficiently mechanistic that causal relationships can be investigated through modeling. They also assume that the laws operate over some geomorphic time and spatial scale that integrates the effects of inherently stochastic and spatially variable processes (although the role of stochasticity is beginning to be explored [e.g. *Tucker* and *Bras*, 2000]). By defining transport laws in terms of specific processes (e.g. creep, landsliding, or river incision) there is an implicit linkage to specific landscape scales. These laws are meant to capture the time-averaged dependency of transport rate on topography for specific processes in which the averaging occurs over time-scales of significant erosion. Processes that build point bars or dunes also are modeled using conservation equations and transport laws, but such features are finer scale and change form over shorter time scales. To emphasize our focus on temporal and spatial scales relevant, for example, to the evolution of drainage basins, we use the term 'geomorphic' transport laws.

Despite the extensive use of geomorphic transport laws, there are significant gaps in our knowledge of them. Only a few studies have been done that provide direct evidence for the form and calibration of these laws. For many geomorphic processes no such studies have been done. These gaps draw into question mechanistic inferences derived from numerical models that are based on unverified or unverifiable transport laws. Underlying this issue is the difference between a transport law and a simple rule. This distinction is not trivial, because it defines the difference between building models upon transport laws that are to some degree testable, and using expressions that provide a desired system behavior, but which cannot be tested.

Here we propose that a geomorphic transport law is a mathematical expression of mass flux or erosion caused by one or more processes acting over geomorphically significant spatial and temporal scales. Such a law is required to solve the conservation of mass equation and is distinguished from a 'rule' because it is mechanistic, and it describes a process that can be observed, parameterized, and verified through field work and laboratory experiments. We recognize that there still are compromises and difficulties in this definition, and we explore these issues below.

Geomorphic modeling is generally directed at interpreting and predicting landscape form and evolution in some tectonic, climatic and lithologic setting. Prediction in the sense of forecasting some future state is not testable, as real-scale landscapes change far too slowly. Recent coupling of laboratory physical modeling and numerical modeling, however, does provide an opportunity to test the predicted evolution of landscapes [*Hancock* and *Willgoose*, 2001; *Lague* et al., 2000a.], and success in this effort would add support to the general approach of geomorphic modeling and the use of geomorphic transport laws. In most cases, however, predicted landscapes are not compared quantitatively with real ones, but instead the behavior and form of the hypothetical landscapes, using specific initial conditions, boundary conditions and transport laws, are explored to give insight about controls on real landscape morphodynamics. The weak and generally untested assumption in this case, then, is that the natural landscape behaves like the hypothetical one. While there are a number of measures of landscape properties, in general we lack a set of metrics that can be used to reject incorrectly formulated models and constrain the correct formulation of geomorphic transport laws (and the insights they provide when used in models). This problem is further exacerbated by the possibility that similar landscape morphology may arise from different processes.

Below we examine many of the questions raised above about prediction and geomorphic transport laws. The central goals of this paper are to: 1) define, by example, the kinds of questions and observations for which the use of geomorphic transport laws seems most appropriate, 2) compare various landscape morphodynamic modeling approaches to illuminate what might be the most appropriate application of geomorphic transport laws, 3) review evidence for various geomorphic transport laws, and 4) suggest, with new high resolution topographic data, how geomorphic transport

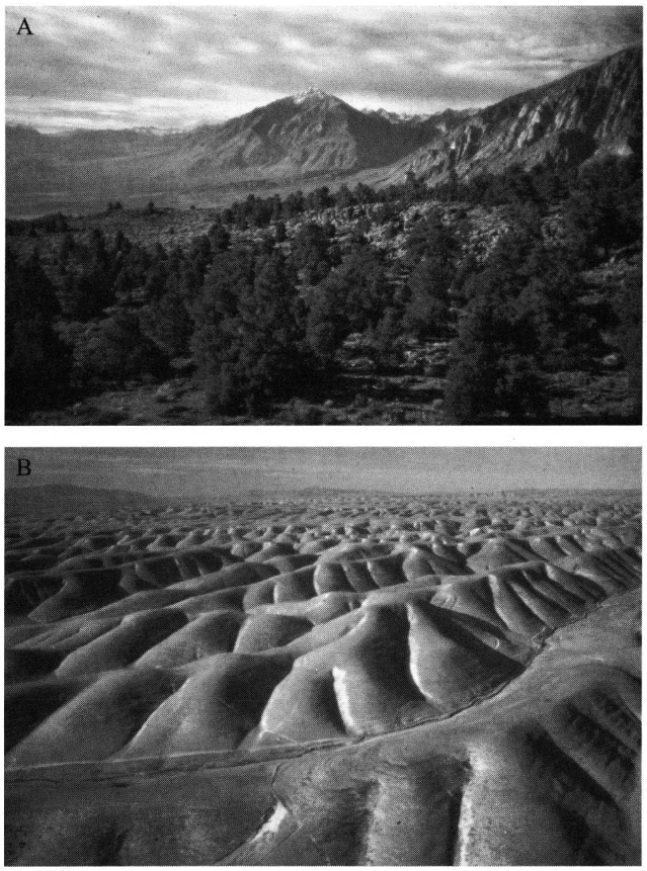

Figure 1. A. View of the eastern side of the Sierra Nevada Mountains, California (Mt. Tom near Bishop). B. Soil-mantled, ridge and valley topography near Salinas, California

laws might be used in numerical models of real landscapes to gain insight into landscape morphodynamics.

2. SOME QUESTIONS THAT MOTIVATE THE DEVELOPMENT OF GEOMORPHIC TRANSPORT LAWS

Figure 1 shows two contrasting landscapes. One is a mountain range, which rises abruptly from the adjacent lowlands to heights sufficient that glaciers have periodically developed and advanced down the valleys. The other is a highly dissected soil-mantled lowland. Below we briefly discuss five questions, motivated by the contrasting morphology of these landscapes, which provide examples of problems that may be best approached through the use of geomorphic transport laws.

2.1 What Controls Relief?

While it is widely recognized that local relief (the height difference between valley bottom and adjacent hilltop) is a distinctive attribute of landscapes, a general theory for predicting relief, based on use of geomorphic transport laws, is lacking. In high relief terrain (Figure 1A) intensive wear by glaciers [e.g. *Brozovic et al.*, 1997] and landsliding due to strength limitation of bedrock [e.g. *Schmidt* and *Montgomery*, 1995] may impose limits. In contrast, neither bedrock strength limitations nor glaciation explains the relief of the landscape shown in Figure 1B. Here soil is transported downslope by shallow mass wasting processes and perhaps overland flow. Channel spacing sets the horizontal length of the hillslope, and channel incision rate sets the pace of hillslope erosion. If incision rates are sustained for a sufficiently long period, there will be a tendency for the hillslope shape to be adjusted such that erosion rate is spatially uniform across the hillslope. The transport process that shapes the hillslope and the intensity of channel incision then set the relief. In the simple case of soil transport proportional to slope, for example, it has been shown that relief varies directly with incision rate and the square of the hillslope length and inversely with the constant of proportionality relating transport to slope [e.g. *Kirkby*, 1971; *Koons*, 1989; *Dietrich* and *Montgomery*, 1998]. Hence, quantification of geomorphic transport laws is crucial to relief prediction.

2.2 Why are Some Landscapes Soil-Mantled and Others Bedrock Dominated?

A striking difference between the two landscapes shown in Figure 1 is the dominance of bedrock on the slopes in Figure 1A and the absence of any bedrock exposure in Figure 1B. This difference matters because the processes responsible for erosion of bedrock hillslopes will differ greatly from those that transport loose soil material downslope. Yet nearly all numerical models apply hypothesized soil transport laws to steep mountainous landscapes where bedrock commonly prevails. We currently lack transport laws for the erosion of exposed bedrock slopes.

Bedrock hillslopes emerge where the potential erosion rate exceeds the production rate of loose debris or soil from the bedrock. Such conditions may be met where channel incision rates or uplift rates are high, or the rate of production from bedrock is low. A soil production law is needed to model the conditions that favor bedrock or soil-mantled conditions, and we discuss such a law below.

2.3 What Controls Drainage Density?

Drainage density (the total length of channels per unit area of landscape) varies greatly among landscapes, and its quantification and prediction should provide valuable

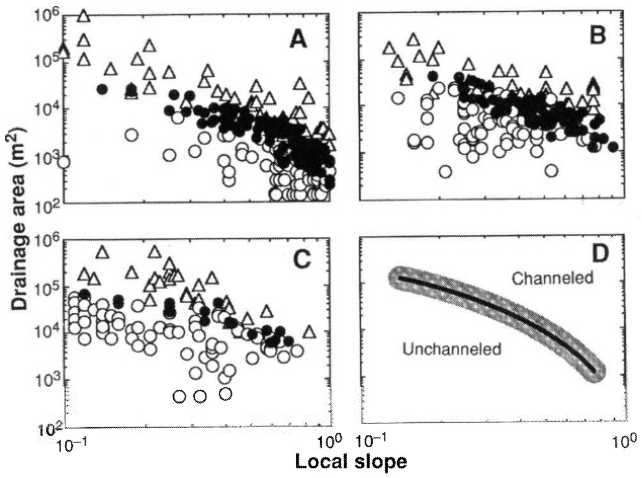

Figure 2. Drainage area and local slope for channel heads (solid dots), unchanneled valleys upslope from the channel heads (circles), and low-order channels (triangles) from: A. coastal Oregon (adjacent to the area represented in Figures 4 and 5), B. northern California, and C. southern California. D. summarizes general finding that there is a area-slope topographic threshold distinguishing channeled and unchanneled areas. This threshold also corresponds to the boundary between hillslopes and channeled valleys (from Montgomery and Dietrich, 1992).

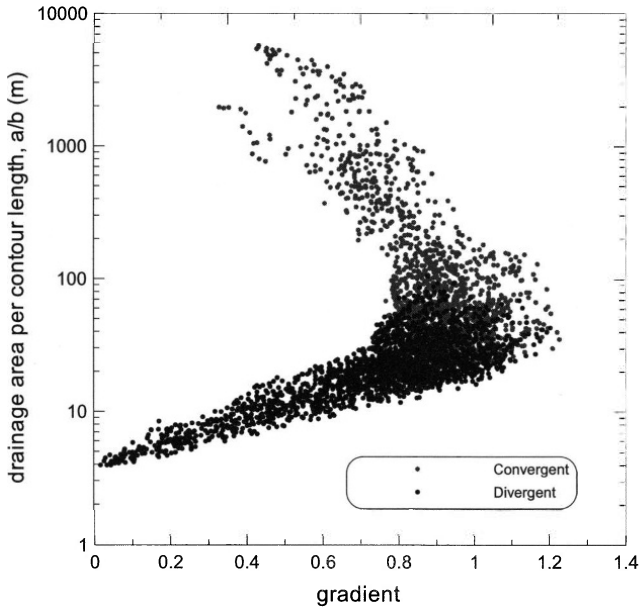

Figure 3. The variation of drainage area per grid cell size with local gradient for an entire small basin. Black dots represent terrain with divergent curvature, which are the hillslopes, and the gray dots are the convergent terrain. Values are calculated from topography gridded to 4m, hence the smallest a/b is 4m (from Roering et al., 1999).

insight about controls on landscape morphology. As noted in the previous section (2.2) channel spacing (the inverse of drainage density and roughly twice the hillslope length) may directly influence local relief. Figure 1B shows that drainage density can be highly regular and that unlike some approximations used in fractal analysis, there is a finite drainage density. Channels do not branch infinitely, but rather there is a finite extent of channelization [e.g. *Montgomery* and *Dietrich*, 1992]. Between channels lie undissected hillslopes, where the smoothing effects of hillslope transport prevail.

Two empirical observations further define the idea of a limit to the drainage density and the distinction between hillslopes and channels. Figure 2 shows a plot of drainage area against slope for channel heads and for local drainage areas above and below the channel head. This suggests that there is a threshold drainage area for a given slope, above which channel incision begins, and if so, the steeper the slope, the smaller the drainage area to initiate a channel, hence the greater the drainage density [*Montgomery* and *Dietrich*, 1988; 1992]. In Figure 3, the entire landscape of a small catchment is depicted in a plot of drainage area per unit contour length (a/b) against local slope. Cells that locally have planform covergence (valleys) are distinguished from those that are divergent (hillslopes). Such plots [see also *Dietrich et al.*, 1992; *Tucker* and *Slingerland*, 1997; *Hancock* and *Willgoose*, 2001] suggest that hillslopes and valleys are created by distinctly different erosional processes with opposing dependencies on drainage area and slope. Topographic gradients tend to increase with increasing drainage area on hillslopes and tend to decrease with increasing drainage area for valleys. Indeed, such plots may provide important clues about dominant erosion processes. Figures 2 and 3 suggest that both erosion thresholds and the relative intensity of valley forming versus hillslope eroding processes determine where channels begin and hence the length scale of hillslopes and the drainage density. Following the pioneering theoretical work by *Horton* [1945] on thresholds to channelization and by *Smith* and *Bretherton* [1972] on the competition of hillslope and erosion processes, several recent theoretical studies have explored mechanisms controlling channel spacing or drainage density [e.g. *Loewenherz*, 1991; *Izumi* and *Parker*, 1995; *Howard*, 1997; *Smith et al.*, 1997a,b; *Tucker* and *Bras*, 1998]. All of these papers point to the need for field verified geomorphic transport laws.

2.4 What Controls Valley Longitudinal Profiles?

River longitudinal profiles, another property that distinguishes landscapes, have a long history of investigation in

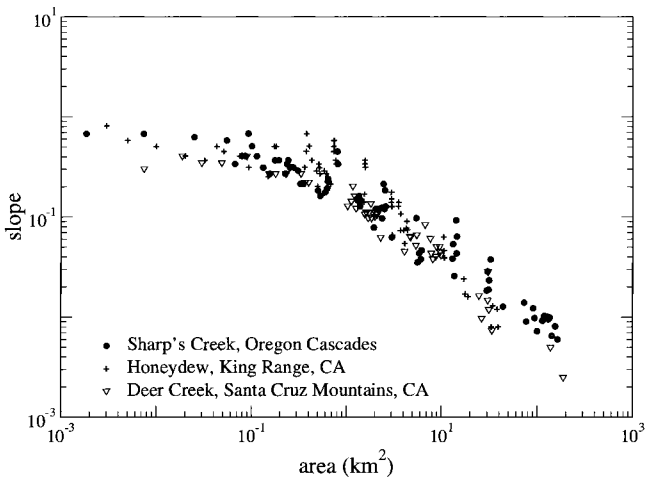

Figure 4. Plot of valley drainage area vs. slope for Sharp's Creek, Oregon, Deer Creek, Santa Cruz Mountains, and Honeydew River, King Range, California. These data, collected by hand from 1:24,000 contour maps, require more than a single power law (*e.g.*, stream power law) because they curve at slopes above ~ 3-10%. Despite large differences in erosion rate between the sandstone–floored rivers, Deer Creek at ~ 0.2 mm/a [*Perg* et al., 2000] and Honeydew at ~ 4 mm/a [*Merrits and Vincent*, 1989], the data define very similar relationships.

geomorphology [e.g. *Gilbert*, 1877; *Davis*, 1902; *Shulits*, 1941; *Mackin*, 1948; *Yatsu*, 1955; *Hack*, 1973; *Ohmori*, 1991]. Not until geomorphic transport laws predicted an inverse relationship between local slope and drainage area [*Howard* and *Kerby*, 1983; *Willgoose et al.*, 1991c; *Seidl* and *Dietrich*, 1992] did this relationship and its controls attract widespread attention [e.g. see references in *Sklar* and *Dietrich*, 1998; *Whipple* and *Tucker*, 1999; *Sklar* and *Dietrich*, 2001]. Explanations for this relationship most commonly assume that transport or channel incision varies with average boundary shear stress or stream power per unit bed area. Bed area depends on channel width, hence wrapped in the explanation of river profiles is also the unsolved problem of what controls the width of channels.

Figure 4 typifies longitudinal profiles of steepland valleys throughout the world [*Stock* and *Dietrich*, in press]. At large drainage area, area-slope data approximate a power law as expected from either sediment transport or bedrock incision varying with shear stress or stream power. With decreasing drainage area, however, the rate of increase in slope declines, leading to a curved relationship on a log-log plot of slope against drainage area. Little work has been published on the factors controlling steep channel slopes at low drainage areas, though empirical evidence points towards scour by episodic debris flows being a primary agent [*Seidl* and *Dietrich*, 1992; *Montgomery* and *Foufoula-Georgiou*, 1993; *Sklar* and *Dietrich*, 1998; *Montgomery*, 2001; *Stock* and *Dietrich*, in press]. Hence, the slope-area plot appears to be a strong indicator of transport or erosion mechanisms. As discussed below, however, different dominant transport or erosion mechanisms may lead to similar results for lower gradient channels where sediment effects and bedrock incision may both matter, making slope-area plots of lower gradient channels less instructive than initially conceived. This, too, points to the need of obtaining experimentally verifiable, mechanistic geomorphic transport laws.

2.5 What Morphologic Properties can be Used to Test Landscape Evolution Models?

Although numerical models of landscape evolution are becoming commonplace, little agreement exists on what topographic measures should be used to compare model and real landscapes. A key issue is what features are a reflection of the erosion mechanisms, i.e. what features distinguish the processes responsible for landscape evolution? As suggested above, relief, bedrock exposure, drainage density, and valley longitudinal profiles all reflect erosion mechanisms. Other measures have been discussed [*Ibbit et al.*, 1999; *Howard*, 1994, 1997; *Densmore* and *Hovius*, 2000; *Li et al*, 2001, *Hancock* and *Willgoose*, 2001]. *Rodriguez-Iturbe* and *Rinaldo* [1997] explore what fractal measures may distinguish real landscapes from model ones.

At a large scale, such as entire mountain ranges (Figure 1A), quantitative comparisons between actual and model landscapes have mostly relied on visual comparison. The most common measure presented has been the cross-sectional profile of the mountain system, with an emphasis on its symmetry relative to tectonic and climate forcing [e.g. *Koons*, 1989; *Willett*, 1999] or the shape of a retreating escarpment [e.g. *Tucker* and *Slingerland*, 1994; Kooi and *Beaumont*, 1994; *van der Beek* and *Braun*, 1999]. Howard [1995] employed discriminant function analysis to distinguish planforms of escarpments associated with different formational mechanisms. *Van der Beek* and *Braun* [1998], *Hurtrez et al.* [1999] and *Lague et al.* [2000b] each review various potential measures and draw different conclusions about topographic attributes that might prove most diagnostic. A consensus should emerge once geomorphic transport laws for mountain systems become well established. Presently, however, such laws are essentially absent.

This section has focused on questions and landscape metrics that lead to development and testing of geomorphic transport laws. Although a number of metrics have been proposed, there is as yet little agreement about how models should be tested and whether these measures are useful. These issues depend in part on the questions driving the

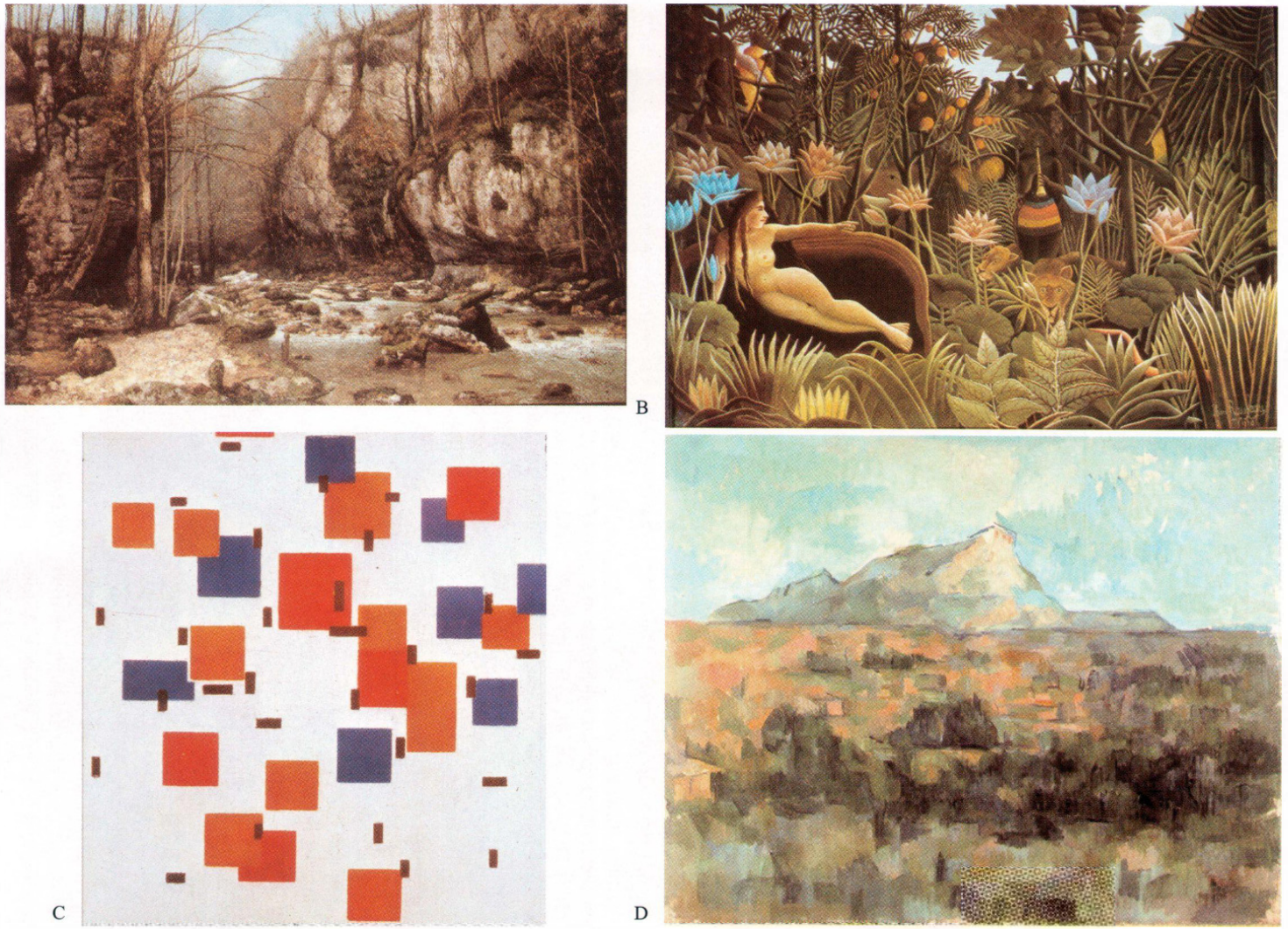

Plate 1. Paintings as metaphors for different approaches to landscape evolution modeling.
A) Detailed realism: G. Courbet, 1868, Streams of the Puits-Noir at Ornans (Norton Simon Museum, Pasadena).
B) Apparent realism: H. Rousseau, 1910, The Dream (The Museum of Modern Art, New York).
C) Statistical realism: Mondrian 1917, Composition in Blue B (Kroller-Muller Museum, Netherlands).
D) Essential realism: Cezanne 1904-1906, Mont Sainte-Victoire Seen from Les Lauves (Private Collection).

modeling. Below we discuss four approaches to modeling that may serve to illuminate this point.

3. FOUR MODELING APPROACHES TO EXPLAINING MORPHOLOGY

A wide range of modeling approaches has been proposed to tackle the problem of predicting landscape form and evolution. Here we identify four general modeling approaches to provide background for the approach that calls for quantifying geomorphic transport laws. All four modeling approaches have utility. The names we use are convenient handles only and are not proposed as terminology to be adopted by others. We give each one the broader label of "realism" because of the common interest in explaining some aspect of real landscapes. This goal is shared with the painters concerned with realism, and we use different landscape paintings as metaphors for different approaches to modeling (Plate 1).

3.1 Detailed Realism

Real landscapes contain a mixture of general trends and site-specific conditions. The painting by Gustave Courbet illustrates a detail rich landscape (Plate 1A). We can see individual moss covered boulders in the bed of the canyon, evidence of lighter colored gravel just poking through the water, fractured bedrock cliffs, and trees at specific locations. An experienced geomorphologist may be able guess the drainage area, bankfull width and depth just from the relative scaling visible in the work. Prediction of such specific properties, especially at particular places and time, is far beyond the capability of any current geomorphic model and would require such detailed knowledge of materials and sequencing of stochastic events (climatic, tectonic and intrinsic) as to be essentially unattainable. We cannot hope to predict highly site-specific conditions over geomorphic time scales. On the other hand, given some information, shorter- term predictions can be reasonably made for some features. For example, with sufficient topographic, sediment load and discharge information one can predict river grain size [e.g. *Parker*, in press], the temporal variation in river bed depth with differential sediment loading [*Parker*, 1991a,b; *Benda* and *Dunne*, 1997; *Cui et al.*, in press; *Parker*, in press], or the migration rate and cross-sectional morphology of river bends (for a given channel width) [e.g. *Ikeda* and *Parker*, 1989]. Some critical features at this shorter and finer time scale, channel width for example, remain poorly explained and lack a general theory.

3.2 Apparent Realism

It is common practice now to use process-based transport relationships in large- scale numerical models to predict landscape evolution [e.g. *Anderson*, 1994; *Tucker* and *Slingerland* 1994; *Kooi* and *Beaumont*, 1994, 1996; *van der Beek* and *Braun*, 1998, 1999]. Often rules are added to account for the hypothesized effect of some process. The coarse grid scale of these computationally intensive models means that the transport equations are applied on scales much greater than the process they are meant to represent. Hence it is difficult to interpret aspects of the model outcomes [*Dietrich* and *Montgomery*, 1998]. Furthermore, although such models produce detailed topography, typically the only testing of the model is whether the outcome looks right, or whether there is a match with the use of coarse measures, such as fractal dimension [e.g. van der *Beek* and *Braun*, 1998]. In some ways this is like the painting by Henri Rousseau (Plate 1B, *The Dream*). The painting is rich with detail showing what appear at first to be real plants, animals and people. But it may be an impossible collection of things having only metaphoric connection with the real world. This is not to say that such models have no value. Because of computational demands, current lack of knowledge about how to scale up finer scale mechanisms, and a lack of quantitative morphology or dynamics data, those models that examine large-scale landscapes are by necessity approximate and create what can be called an apparent realism. Insight may nonetheless be gained about possible linkages between uplift, erosion and topography at such a very coarse scale.

3.3 Statistical Realism

Some have argued that the essential goal in geomorphology should be an understanding of the most general emergent relationships that a self-organized system produces [e.g. *Leopold* and *Langbein*, 1962; *Rodriquez-Iturbe* and *Rinaldo*, 1997]. Such features would be shared widely by landscapes of varying climate, bedrock and tectonic regime. It follows that if such features exist, then the detailed mechanics of the processes, which would also vary among these different landscapes, should not be important. Instead certain principles, perhaps having to do with energy expenditure and space filling limitations, or the commonality of the mathematical form of erosional processes, can be identified and shown to explain these emergent features. Mondrian's Composition in Blue B (Plate 1C) offers a model of landscape elements (although the artist intended the work to be about art itself not about any particular real subject). No recognizable landscape is present, but these

elements are arranged in space relative to each other in a manner that suggests some organizing principle. Statistical or mathematical analysis can be used to quantify this pattern, thus defining a statistical realism. Rules-based models can then be explored to see what produces these patterns [e.g. *Chase*, 1992; *Rigon et al.*, 1993; *Veneziano* and *Neimann*, 2000a,b]. The book by *Rodriguez-Iturbe* and *Rinaldo* [1997] persuasively lays out this argument and analysis in a thoughtful and thorough manner.

3.4 Essential Realism

Real landscapes evolve in the four dimensions of spatially varying material properties and boundary conditions with temporarily varying external driving forces. This condition combined with non-linear, threshold dependent erosion processes leads to a significant component of indeterminacy in the evolving topography. Therefore, it is unrealistic to expect to predict the exact topography of a landscape at any particular time, including the present. Instead the gross trends, the quantitative relationships, such as illustrated in Section 2 and the references cited therein, are the features landscape evolution models can realistically hope to explain. Cezannes's Mount Sainte-Victoire (Plate 1D) shows the essential elements of the landscape: the outline of the mountain rising above a surrounding plain, and houses clustered amongst trees in the foreground. This is a recognizable specific place, but only the most general features are identified. Such a view overlaps with the goals expressed in the "statistical realism" approach outlined above. Major differences exist, however. The essential realism approach considered at length below holds that mathematical expressions of transport and erosion (geomorphic transport laws) can be identified with observable processes in the field, that these expressions can be parameterized from field measurements, not just tuned in a model, and as such, these models can be tested and rejected if they fail to predict observed phenomena, rather than simply retuned to the desired result. The "statistical realism" view is not concerned with capturing real processes with field-based parameters, and its focus is more on the general common elements amongst diverse landscapes. In contrast, the goal of the "essential realism" approach is the explanation of the differences among landscapes [e.g. *Howard*, 1997]. This approach also differs from the "detailed realism" focus, which requires considerable information about materials, climatic properties and the like, and must therefore be parameter-rich and of limited explanatory power in both space and time. As mentioned above, the "apparent realism" approach has tended to use geomorphic transport laws but ignored the scale limitations of these expressions, leading to misapplication and unclear conclusions about landscape form.

4. GEOMORPHIC TRANSPORT LAWS

Here we review current evidence for geomorphic transport laws. This evidence can come in the form of calibration of model parameters from field measurements at the scale at which the processes occur or from physical modeling studies. It is crucial that transport or erosion equations be directly tested independent of any landscape model. Without this ability, the only way to examine model performance is to examine the predicted landscape morphology and evolution. Testing only the outcome of the model, rather the components of the model that led to the prediction, gives limited insight about causality. The discussion below shows, however, that it is difficult to quantify directly process rates relevant to geomorphic time-scales. There are many knowledge gaps and few studies upon which we can rely. Other approaches may be fruitful. By analogy to geophysical investigations that use seismic and gravity data to characterize crustal structure, the systematic application of inverse methods may enable geomorphologists to use erosion rate and topographic data for the calibration of geomorphic transport laws [e.g. *Parker*, 1994].

4.1 Conservation of Mass Equation and Geomorphic Transport Laws

Landscapes are displaced both vertically and horizontally by tectonic deformation [e.g. Willett, 1999] and are eroded primarily by mass-wasting processes, fluvial entrainment and wear, and in some climates, by various ice-related processes. Mechanical and chemical breakdown reduce the strength of bedrock and produce erodable material, but may also enhance the resistance to erosion through the formation of chemical precipitates in the soil (e.g. calcretes, silicretes, ferricretes and other such chemical precipitates). Formation of these resistant weathering products is not considered here.

In general, we can write the conservation of mass equation for a soil or sediment mantled landscape underlain by bedrock (assuming constant bulk density for simplicity), as

$$\frac{\partial z}{\partial t} = U - P + \frac{\partial h}{\partial t} \tag{1}$$

in which z is the elevation of the ground surface, h is the soil or sediment thickness, P is a sediment production term, and U is the rock uplift rate (Figure 5). P is equal to the soil production rate (conversion of bedrock to soil) on hillslopes

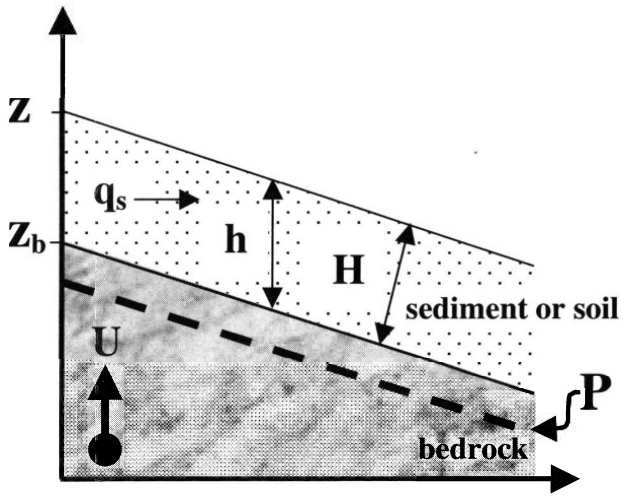

Figure 5. Cartoon illustrating terms in conservation of mass expression (equation 1) for a soil-mantled hillslope or sediment-mantled channel with a ground surface elevation of z. The soil or sediment thickness is h, and uplift, U, is shown as positive upward. Dashed line below soil-bedrock interface illustrates the depth of bedrock converted to soil over a given time period (e.g. soil production rate, P). The sediment transport per unit hillslope width is q_s.The normal depth, H, and the elevation of the soil-bedrock boundary, z_b, used in equation (11) are also shown.

and is equal to the wear or incision rate by concentrated flows in channels. Note that P is always positive. Application of equation (1) differs, however, for hillslopes and channels, as discussed below.

On hillslopes we can write (still assuming no bulk density change for convenience) that the divergence of the sediment transport vector, $\tilde{q_s}$ is linked to storage and production as

$$-\nabla \cdot \tilde{q_s} = \frac{\partial h}{\partial t} - P \tag{2}$$

or

$$\frac{\partial h}{\partial t} = P - \nabla \cdot \tilde{q_s} \tag{3}$$

The transport divergence and production terms are driven by physical forces but are not independent of the soil thickness. For example, the production rate will vary with the thickness of the overlying soil [*Heimsath* et al., 1997], as discussed below.

Substituting equation (3) into equation (1) we get the familiar mass conservation equation widely applied in landscape modeling

$$\frac{\partial z}{\partial t} = U - \nabla \cdot \tilde{q_s} \tag{4}$$

Mass loss through dissolution is not explicitly considered here. It will effect the bulk density terms (which will therefore differ for bedrock lowering, uplift and transport terms). Solution loss may also lead to collapse, in which case it would need to be treated as a separate expression in equation (4). Such morphologic effects of solution appear to be only important in certain rock types (e.g. limestones) or under low uplift and erosion rates.

In most cases the limiting effects explicit in equation (2) are not applied; that is, current models do not account for the fact that the divergence of sediment transport cannot exceed the available storage change plus the local soil production. This is an important limit, however, in most landscapes. If soil production is included (i.e. equation (2)), then where bedrock emerges on the hillslope, the divergence term must equal only the production rate and nothing more, no matter what the potential transport would be if the hillslope were soil mantled. When this limit is applied in a numerical model it introduces a significant unknown: the travel distance per unit time step of sediment across exposed bedrock.

In both the mass transport on hillslope case and the fluvial (surface wash or channel incision) case, bedrock can be eroded beneath a thickness of detached material. But important differences exist because it is often assumed in the fluvial case that bedrock detachment or wear does not contribute significantly to the sediment storage term ($\partial h/\partial t$), hence the production and transport terms in equation (2) become decoupled. Furthermore, in applying the mass conservation equation in river channels, there is a significant fraction of sediment delivered to the channel that plays no role in either sediment storage or bedrock wear. This is the wash load and perhaps most of the suspended load, and it commonly constitutes a large fraction of the total load [e.g. *Nordin*, 1985]. This means that some theory is also needed to determine what fraction of the total load is bedload (and the loss to suspended and wash load during transport needs to be tracked). With these limitations in mind, we can write

$$\frac{\partial h}{\partial t} = -\nabla \cdot \tilde{q_s} \tag{5}$$

for the sediment storage on the channel bed, and

$$P = f(A, S, Q_s, D, \sigma..) \tag{6}$$

for the incision or wear of the bedrock surface. Hence, according to equation (1)

$$\frac{\partial z}{\partial t} = U - f(A, S, Q_s, D, \sigma...) - \nabla \cdot \widetilde{q_s} \tag{7}$$

As in the hillslope soil transport case, the divergence of sediment transport cannot exceed the sediment thickness available in a given time step. P in this case is the erosion rate into the underlying bedrock or regolith. *Gilbert* [1877] referred to this erosion as "corrasion" and it depends on hydraulics represented as a function of discharge or drainage area (A) and slope (S), sediment supply (Q_s), grain size (D) and bedrock strength (σ). Other factors may matter, hence the suggestion of other variables not listed in (6). Commonly it is assumed that the divergence of sediment transport term does not apply to mountain streams, but equation (7) is the more general expression and the limiting effects of transport of coarse sediment may dominate even in bedrock channels [*Sklar* and *Dietrich*, 1998, 2001; *Howard*, 1998].

Three conditions may exist in landscapes that limit the rate of sediment outflow from a landscape. Actively uplifting landscapes are commonly a mixture of all three conditions. In the rare case in which the landscape is fully soil mantled and the channels are covered by sediment at all times, equation (3) applies without constraints and this condition has been called "transport limited" [e.g. *Kirkby*, 1971; *Carson* and *Kirkby*, 1972; *Howard*, 1994]. The rate of removal of sediment is driven by transport capacity and is not limited by supply. If erosion on hillslopes causes bedrock to emerge, the erosion rate becomes limited to the production rate, P, and this condition is referred to as "weathering limited" [*Kirkby*, 1971; *Carson* and *Kirby*, 1972; *Anderson* and *Humphrey*, 1989; *Howard*, 1994]. If erosion by flows (water, ice, or sediment mixture) thins or removes the sediment mantle sufficiently so that the bedrock is subject to wear (not just to weathering), the erosion becomes limited to the wear rate by the flow (Gilbert's "corrasion"). Overland flow erosion of hillslopes may also be limited by its ability to entrain resistant regolith. *Howard* [1994] has called these conditions "detachment limited". *Kirkby* [1971] proposed the term "erosion-limited" to convey a similar idea that erosion is less than the transport capacity and that the erosion rate would be a function of the difference between the transport capacity and the actual transport rate.

Recent work demonstrating that fluvial bedrock wear rate depends on supply and grain size [*Sklar* and *Dietrich*, 2001] and that bedrock incision may only require a minor increase in slope beyond that necessary to transport the supplied coarse load [*Sklar* and *Dietrich*, 1998; *Sklar* and *Dietrich*, 2001] blurs the distinction between 'detachment' and 'transport' limited conditions. The term 'detachment limited' may best apply to waterfalls and steepened reaches on hard rocks carrying minor amounts of coarse sediment, or on landscapes where overland flow erosion cuts rills into cohesive materials, as argued by *Howard* [e.g. 1994, 1997]. The exposure of bedrock and the influence of its material strength only becomes limiting when it forces a different morphology or a different erosion rate.

In order to solve the above equations, besides definition of initial and boundary conditions, mathematical expressions are needed for sediment transport, $\widetilde{q_s}$, and sediment production or fluvial detachment rate, P. Following the discussion above about "laws" and "rules," we argue that the preferable expressions for these two terms would be geomorphic transport laws. Though expressions for P define detachment rate (L/T) rather than a transport rate (L^2/T), for simplicity of language we include them in the "transport law" category. Wear is a form of transport because mass is moved from a stationary reservoir to a mobile form, from one component of the sediment budget to another.

We hypothesize that geomorphic transport laws can be reliably quantified, and that they may possess some "universal" qualities that allow them to be used in diverse landscapes, under boundary conditions and external forcing that differ from the conditions under which they were parameterized. It follows that reasonable explanation of observed features with these laws would then permit numerical model explorations of the interactions of processes under varying driving conditions and materials properties. We acknowledge that geomorphic transport laws are not fully mechanistic, in that they are not always derived from first principles and that they may tend to smear the effects of many processes into single expressions (e.g. representing the transport consequences of tree throw and animal burrowing with a slope -dependent transport law). It is this trade-off of first principles for approximate mechanistic expressions, we propose, that reduces the parameter numbers to the level where they might be fully determined from field measurements or physical modeling experiments. This compromise, however, in most cases, makes the parameters model-dependent.

4.2 Current Knowledge About Geomorphic Transport Laws

Here we review studies that purport to quantify geomorphic transport laws through field measurements or physical experiments. We focus on studies that provide calibration of parameters in transport laws that can be used over geomorphically significant space and time. The combined recent development of high resolution digital elevation data, to quantify topography, and cosmogenic radionuclide dating, to quantify rates of erosion and transport, now offer the promise that there will be significant advances in this area. The processes discussed below are not an exhaustive list, but rather consist of dominant processes that have received the most attention.

4.2.1 Transport of soil by slope-dependent processes. The occurrence of rounded, convex hilltops in badlands puzzled *Gilbert* [1877] until *Davis* [1892] commented that repeated dilation and contraction of loose debris on an inclined surface will induce a creeping, downslope transport, and that the effect of this movement in shaping hillslopes is likely to dominate on divides, where surface wash is not concentrated. *Gilbert* [1909] subsequently pointed out that hilltop convexity is common (not just in badlands) and reasoned that creep resulted from disturbance by expansion and contraction, due to freeze-thaw, wet-dry and hot-cold cycles, and biologic activity. He proposed that these processes varied with slope and that a hilltop undergoing steady state erosion should consequently have a convex form. *Culling* [1960] formalized this hypothesis by solving one-dimensional mass conservation equations using a transport law that assumed flux proportional to local hillslope gradient. Subsequently many researchers explored the role of boundary conditions such as channel incision rate and expanded the application of this law into many geomorphic settings, including modeling entire mountain ranges [see references in *Fernandes* and *Dietrich*, 1997; *Martin*, 2000].

This transport law remains the most commonly used expression to predict hillslope evolution in numerical models. Written most simply, it is

$$\widetilde{q_s} = -K\frac{\partial z}{\partial x} \tag{8}$$

in one-dimension, or more generally

$$\widetilde{q_s} = -K\nabla z \tag{9}$$

in which $\widetilde{q_s}$ *is* the volumetric sediment transport per unit contour length ($L^3/L\text{-}T$), $-\partial z/\partial x$ is the local hillslope gradient, z is local elevation, x is distance from the divide, and K is constant of proportionality with units like that of a diffusion coefficient (L^2/T). Few studies, however, have attempted to collect field data to determine if the linear flux law applies and what the appropriate value of the diffusion coefficient may be, especially for geomorphically significant time periods. Note, too, that the bulk density term is absent in (8) and (9). This term will depend on solute losses and strain (either expansion or collapse) associated with weathering and biologic activity. *Martin* [2000] provides a list of references that report measurements of short-term (up to decades) creep rates. Most studies have assumed that equation (9) is correct and used the evolution of dated escarpments to estimate the diffusivity coefficient [e.g *Hanks* and *Wallace*, 1985; *Avouac* and *Peltzer*, 1993; see summary in *Fernandes* and *Dietrich*, 1997]. Moving a step closer to testing equation (9), *McKean* et al. [1993] used rates of colluvium accumulation in hollows and local hillslope gradients (reported by *Reneau* [1988]) to calculate an average diffusivity coefficient of 49 ± 37 cm^2/yr for 34 sites in California, Oregon, and Washington. *Fernandes* and *Dietrich* [1997] used an estimated long-term erosion rate, based on colluvium accumulation rate in a hollow and the curvature of the adjacent side slope to estimate the diffusivity coefficient.

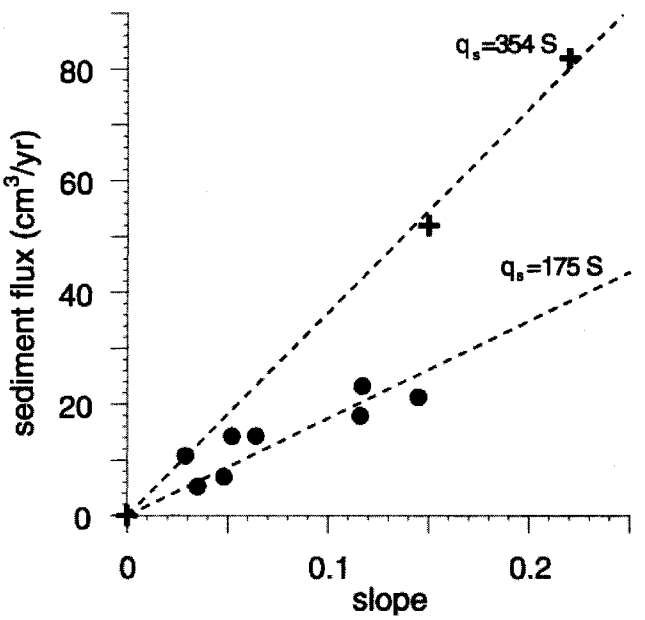

Figure 6. Plot of slope versus soil flux rate for sites in a clay-rich soil in coastal California (crosses; from *McKean et al.*, 1993) and an alpine hillslope in the Wind River Range, Wyoming (infilled circles; from *Small et al.*, 1999). Data from both sites (acquired using cosmogenic radionuclides)

The only direct confirmation of the slope dependent transport hypothesis are the two studies employing cosmogenic radionuclide dating [*McKean* et al., 1993; *Small* et al., 1999], Figure 6. These two studies focused on very different landscapes. The *McKean* et al. [1993] site is formed on a over-consolidated marine shale near San Francisco, California, that weathers to a high-plasticity clay which is subject to seasonal cycles of wetting, shear flow followed by drying and cracking [e.g. *Fleming* and *Johnson*, 1971], and some biogenic mixing. The *Small* et al. [1999] site is on a summit flat in the Wind River Mountains where the granitic soils are probably moved by frost creep [*Anderson*, 2002]. Both sites have gentle slopes not exceeding 23% even 100 m from the ridge.

These two tests strongly support the application of the simple linear diffusion transport model on low gradient soil or regolith mantled hillslopes where surface wash is insignificant. Three issues, however, persist. First, the physical basis of equation (9) is not well established. Simple geometry suggests that if dilation is normal to the surface and contraction is vertical, then steeper slopes should produce greater the displacements. *Furbish* and *Dietrich* [2000], however, have proposed that the slope dependency results from a vertical decrease in porosity with depth below the surface, which leads to a horizontal component of increasing porosity on inclined dilating soils. Soil particle fluxes that tend to loft soil are balanced by gravitationally-driven settling of particles into available pore space, such that greater pore space gradients enhance settling, hence transport. Equation (9) obscures the importance of soil depth and sheds no light on the vertical variation in soil transport velocity. *Furbish* and *Dietrich* [2000] argue that the diffusion coefficient includes the influences of the active soil thickness, particle size, porosity structure and frequency of dilational activity with depth. *Anderson* [2002] has proposed that transport due to frost creep is depth independent in thick soils, but decreases where soil thins and goes to zero in the absence of a soil mantle.

A second issue regarding equation (9) is that it appears to have narrow applicability. As discussed in the next section, it does not apply to steep slopes. It also does not apply where bedrock emerges at the surface. These two limitations make it inappropriate to apply this equation to mountainous landscapes, or to any landscapes where slopes greatly exceed 20%.

The third issue concerns the application of equation (9) in numerical models of large areas. These numerical models may have grid sizes that are equal to or greater than hillslope lengths and are meant to represent steep landscapes were bedrock outcrops are common. Typically the transport coefficient, K, is treated as a parameter and simply

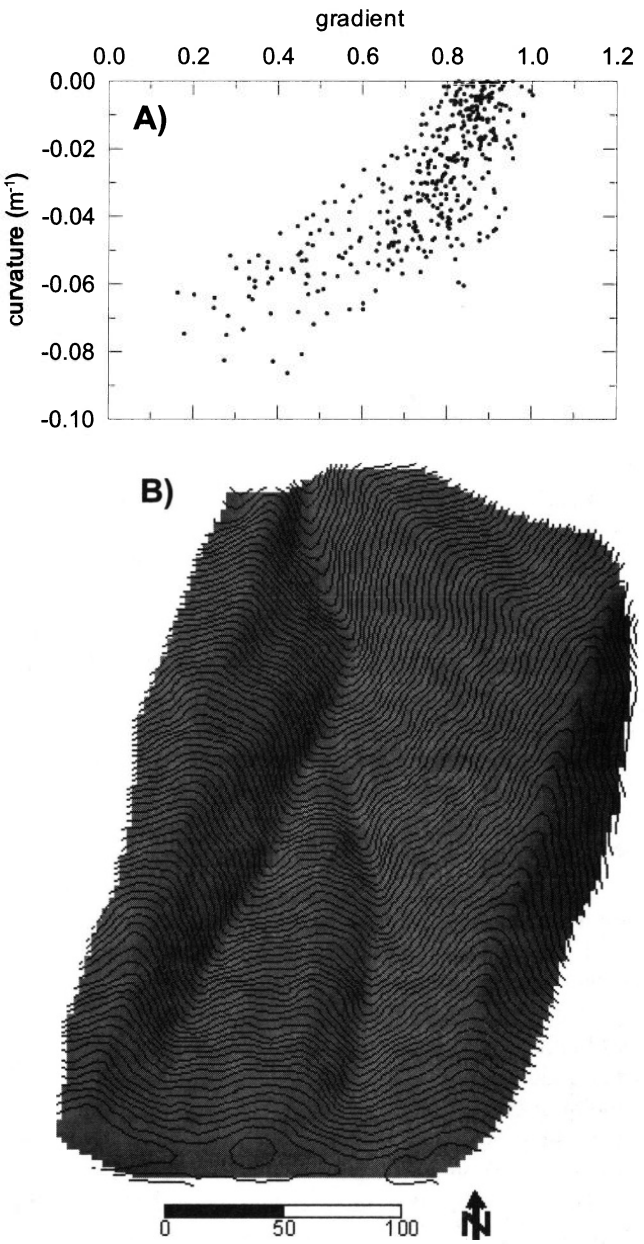

Figure 7. Topographic characterization of a small basin in the Oregon Coast Range where airborne laser swath mapping provided data with an average density of 2.3 m. A. Local curvature versus hillslope gradient for area shown in map. B. 2.5 m contour map used to generate plot in A. Details of procedure are reported in *Roering et al.* [1999]. Low gradient sites, typically at or close to the ridge top, tend to be highly convex (negative curvature). Convexity declines progressively downslope as gradient increases.

increased (often by orders of magnitude) to reach the desired erosion rate [e.g. *Koons*, 1989; *van der Beek* and *Braun*, 1998; *Hurtrez* et al., 1999]. Applying this transport law in this way violates the clear scale and process dependent properties of the law, rendering interpretation of the model results difficult [*Dietrich* and *Montgomery*, 1998]. If it can be shown through field measurements that slopes and drainage areas of coarse grids (cell size larger than a small fraction of the fundamental hillslope length) accurately portray the erosion and sediment fluxes with a scaled up diffusion, then application in this way would have greater relevance to real landscapes.

4.2.2 Non-linear mass transport. In steep landscapes, hilltop convexity tends to be confined to a narrow, low gradient area near the divide. With steepening gradient further downslope, hillslopes tend to become less curved to nearly straight (Figure 7). Hypothesizing that this change in morphology was due to the onset of shallow landsliding on steeper slopes, several researchers [e.g. *Kirby*, 1984, 1985; *Anderson* and *Humphrey*, 1989; *Anderson*, 1994; *Howard*, 1994a, 1997] proposed sediment transport expressions that vary nonlinearly with gradient. *Andrews* and *Bucknam* [1987] argued from a theory for ballistic particle transport along slopes that a non-linearity should exist. *Roering* et al. [1999] proposed that the balance of frictional and gravitational forces in a soil undergoing disturbance-driven transport can be used to quantify transport rates. Net downslope transport is calculated as the difference between upslope and downslope transport components. Whereas upslope transport is resisted by gravity and friction, downslope transport is resisted by friction, but aided by gravity. This analysis led to

$$\widetilde{q_s} = \frac{K_{nl} \nabla z}{1 - \left(\frac{|\nabla z|}{S_c}\right)^2} \qquad (10)$$

in which $\widetilde{q_s}$ is the volumetric sediment transport vector, z is the surface elevation, S_c is the effective coefficient of friction and K_{nl} is a transport coefficient (equal to the ratio of power expenditure per unit area to the product of soil bulk density, coefficient of friction squared and gravitational acceleration). According to equation (10), sediment flux is proportional to slope at low gradients and becomes strongly non-linear as it approaches the threshold slope, S_c. Though derived by a different set of assumptions, this equation is identical to that proposed by *Andrews* and *Bucknam* [1987], and gives a similar slope dependency as Anderson and *Humphrey* [1989] and *Howard* [1994]. *Roering* et al. [1999] did not propose equation (10) as a landslide sediment

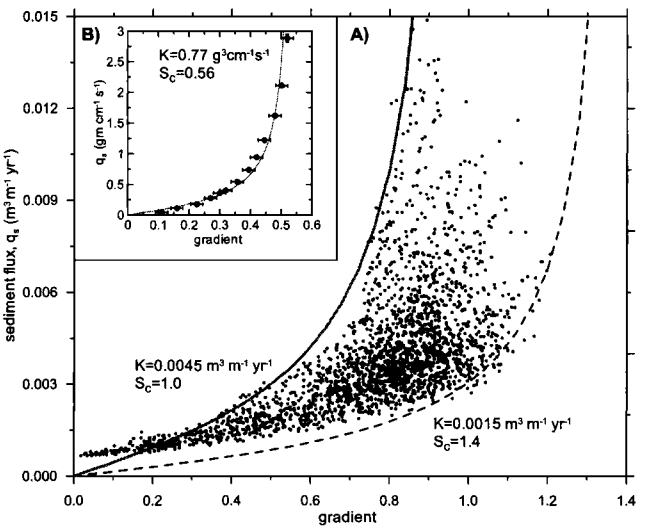

Figure 8. A. Plot of sediment flux versus local hillslope gradient derived from high resolution topographic data in the Oregon Coast Range [from *Roering et al.*, 1999]. Parameters reported are best-fit estimates of equation 10 in the text. B. Measured sediment flux versus gradient derived from acoustically-disturbed sandbox experiments [*Roering et al.*, 2001]. Line passing through data is for the best fit values of equation 10.

transport law, but instead they found that dilational disturbances on hillslopes will intrinsically cause non-linear transport [*Roering* et al., 2001b]. The success of the linear theory reported by *McKean* et al. [1993] and *Small* et al. [1999] is consistent with the non-linear theory because for low gradient slopes equation (10) predicts an approximately linear dependency.

Roering et al. [1999] combined high-resolution digital elevation data (obtained from airborne laser swath mapping) with longer-term erosion rates determined from cosmogenic radionuclide dating [*Heimsath*, 2001] to calibrate equation (10) (Figure 8). Subsequently, *Roering* et al. [2001b] conducted laboratory experiments that provided strong evidence in support of their nonlinear sediment transport theory. Furthermore, they discovered that sediment transport systematically changed from granular creep at lower slopes to episodic landsliding as the slopes approached the critical value, S_c (Figure 8). This suggests that equation (10) may serve as a transport model for the full range of granular transport on hillslopes (from creep and biogenic transport to shallow landsliding) in which excessive pore pressures are not important. From these analyses and from numerical modeling, *Roering* et al. [2001a,b] conclude that once slopes are sufficiently steep that the sediment transport becomes strongly nonlinear, hillslope relief and slope angle

are not sensitive indicators of erosion rate. The time-scale of morphologic adjustment to changing boundary conditions under non-linear transport is also predicted to be considerably faster than linear transport.

Non-linear transport has been inferred from empirical observations of transport. *Gabet* [2000] estimated the displacement of soil caused by gophers over a four-month period. He found that surface transport distance increased with local hillslope gradient and argued that an empirical third order polynomial fit the data best, giving a non-linear increase in transport as the flux rate approached that of the angle of repose of the loose displaced sediment. Given the short period of the measurement, it is difficult to evaluate whether his results are meaningful over geomorphic time scales. Furthermore, the equation chosen to fit the data has no physical basis and therefore may be of limited utility. Some other short-term field measurements of fluxes have suggested that rainsplash is a non-linear transport process [e.g. *Mosely*, 1973; *Moeyersons* and *De Ploey*, 1976]. *Martin* [2000] showed plots of estimated rates of landsliding (from aerial photographs) and basin gradients (although it is not clear how these were calculated) to which she fit hyperbolic tangent functions, which exhibit an odd dependency on slope at high slope values. In this case it is not clear whether these short-term rates are representative of geomorphically significant periods, and furthermore this fit may tend to mix various processes, for example creep-biogenic transport and rapid granular displacement due to elevated pore pressure driven landsliding. A strength of Martin's approach, however, is the attempt to make flux measurements related to slope at topographic scales often employed in modeling. *Riebe* et al. [2000] report power law relationships between erosion and slope based on cosmogenic radionuclide measurements of sand from small catchments. These relationships record the effects of a mixture of all hillslope transport processes, and thus cannot readily be translated into an evaluation of a particular transport law.

As in the linear transport case, the application of a non-linear transport law implicitly assumes a transport-limited, soil mantled landscape. Bedrock will tend to outcrop where slopes approach the threshold value, hence further constraints on the dynamic response of landscapes occur, and additional erosional or transport mechanisms become significant. Also, as before, the non-linear transport law (equation 10) is strongly scale dependent. If it is used in a coarse grid model on length scales equal to or greater than hillslope lengths, the results may have limited bearing on real landscapes.

4.2.3 Soil production. For hillslope sediment transport to occur, unconsolidated surficial material must be available for transport. As discussed above, thickness of soil is controlled by the balance between the divergence of sediment transport and the conversion rate of bedrock to soil, P, (equation 3). Hence, soil production is part of the mass balance determining soil depth (equation 4), and it is appropriate to think of expressions for P as a form of transport or erosion law. Here we use the term "soil production function" as *Heimsath* et al. [1997] proposed, to refer to mathematical expressions for the rate of conversion of bedrock to soil. A distinction is made between soil, defined here as material lacking relict rock structure, and the underlying weathered rock and fresh bedrock. Saprolite, the most weathered state of bedrock, refers to soil-like material that retains rock structure. This distinction based on relict rock structure is made because it distinguishes material that has physically moved from that which has not, which is the key distinction in defining bounds of transport processes. Furthermore, we propose to distinguish the process of weathering from soil production. Weathering alters the state of material, but does not necessarily cause disruption of the bedrock; saprolite is a clear example of this. Bedrock disruption is perhaps most commonly caused by biogenic activity, with the frequency of disruption diminishing as the thickening of soil reduces the probability of the activity reaching the bedrock. Abiotic processes include the host of mechanical and chemical processes that disintegrate and disrupt the bedrock, such as freeze-thaw, wetting and drying and severe dissolution losses. The term regolith is an alternative word for what we call soil here, but this term includes surface fragmental material of any origin [*Bates* and *Jackson*, 1984] and may not clearly distinguish soil from saprolite.

Gilbert [p.103, 1877] first recognized that the rate of soil production (he used the term weathering) should depend on the thickness of the accumulated disintegrated rock. He proposed that the production would cease under thick deposits and increase with progressively thinner deposits, but that once bedrock emerged at the surface, production rate would plummet. *Carson* and *Kirkby* [1972] popularized this inference into the well-known landscape distinction of being either "transport-limited" (where production can keep pace with transport and there is a soil mantle) versus "weathering-limited" (where bedrock emerges and erosion is limited to rate of conversion of bedrock to soil). They and many others [e.g. *Ahnert*, 1967; *Cox*, 1980] have hypothesized that the peak production rate may occur at some shallow soil depth, giving the soil production function a non-monotonic relationship with soil depth. Until recently, however, the soil production function remained unquantified.

In a series of papers, Heimsath and colleagues have reported the use of cosmogenic radionuclide measurements

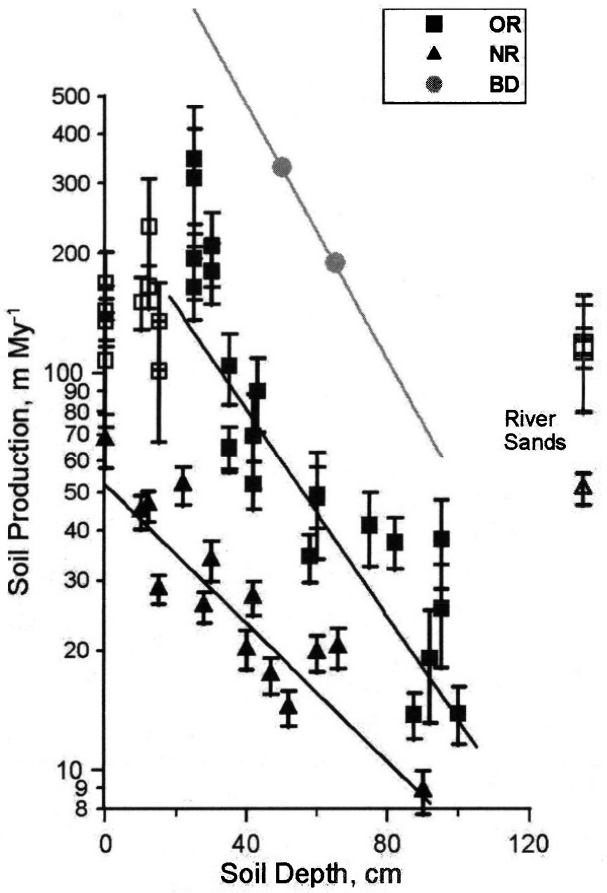

Figure 9. The soil production functions for three sites. These data bracket the entire data set of 7 sites reported in Heimsath [1999]. The Nunnock River area (NR), southeastern Australia [*Heimsath* et al., 2000], had the lowest production rate for a given soil thickness, with $\varepsilon_0 = 53$ m/my, $\alpha = 0.02$ (1/m). Sites at Point Reyes, California [*Heimsath* 1999], $\varepsilon_0 = 81$m/my, $\alpha = 0.016$ (1/m) and Frogs Hollow, southeastern Australia [*Heimsath* et al., 2001b], $\varepsilon_0 = 141$, $\alpha = -0.042$, lie between the Nunnock River and Oregon Coast Range [*Heimsath* et al. 2001a], $\varepsilon_0 = 268$, $\alpha = 0.03$. Data from the San Gabriel Mountains were similar to the Oregon Coast Range, i.e. $\varepsilon_0 = 318$, $\alpha = 0.038$. Oregon (OR) production rates for shallow samples and exposed bedrock samples (open squares) were not used in the regression. See text for further discussion. The soil production rates at all sites except Black Diamond Mine (BD) were determined by using concentrations of in situ produced cosmogenic nuclides as described in Heimsath et al. [1999]. The two values at BD were derived from garden variety ^{10}Be in soils [*McKean* et al., 1993] and suggest $\varepsilon_0 = 2078$, $\alpha = 0.037$.

and the mapping of the topographic controls on soil depth to quantify the soil production function [*Heimsath*, et al., 1997; 1999; 2000, 2001a,b; *Heimsath*, 1999]. They found that in seven different environments, ranging from the steep, highly dissected topography of the Oregon Coast Range underlain by sandstone to the rolling granite hills of the southeastern Australian highlands, production rate declines exponentially with increasing soil thickness (Figure 9).

Writing P as $-\partial z_b/\partial t$, the soil production function is

$$-\frac{\partial z_b}{\partial t} = \varepsilon_0 e^{-\alpha H} \tag{11}$$

in which z_b is the elevation of the soil-bedrock interface, is the production rate of exposed bedrock ($H = 0$), H is the soil thickness normal to the ground surface (see Figure 5) and a α is a parameter (1/length). As Figure 9 shows, maximum production rate ranged from 52 to 2078 m/My, whereas the rate of decline (α) had a narrow range from 0.02 to 0.042 (1/m). The peak erosion rate appears to vary with both rock type and degree of weathering of that rock type. The highest erosion rate occurred in the over-consolidated marine shales studied by *McKean* et al. [1993], which disaggregated with the addition of water, whereas the lowest rate occurred on the least weathered site, fractured granite in southwestern Australia [*Heimsath* et al., 2000]. Of the five cases where the full range of soil depth and underlying bedrock were sampled, two showed evidence of a peak production rate at a depth greater than zero. In the Frogs Hollow case [*Heimsath* et al., 2001], peak production occurred at 25 cm. As Dietrich et al. [1995] pointed out, in such a case there should be no soil thickness found between 25 and 0 cm because thin soils are unstable to erosion perturbations (leading either to thicker soils or exposed bedrock). *Heimsath* et al. [2001] were indeed unable to find any soil coverage between zero and 25 cm. In the Oregon Coast Range case, *Heimsath* et al. [2001] found that under soils thinner than that associated with the peak production rate, the bedrock appeared less weathered. Because thin soil depths were common rather than absent, they inferred that the decline in production rate under shallow to absent soil reflected the less weathered and presumably more resistant bedrock. In a related study, *Small* et al. [1999] reported that the production rate beneath 90 cm of soil was twice that of exposed bedrock, providing further support for a peak production rate at a finite soil depth. *Small* et al. [1999] also correctly point out that the determination of the production function in equation (11) using cosmogenic radionuclides requires accounting for the effects of dissolution on quartz concentration in the soil. This effect is only significant in strongly weathered soils. For the purposes of landscape evolution modeling, it appears that the primary consequence of having a peak in production rate at a finite

soil depth versus at exposed bedrock is the effect this would have on predicted patterns of soil depth [e.g. *Dietrich*, et al., 1995]. The larger effect of including soil production in landscape models is the prediction that bedrock will emerge where local erosion rates exceed the peak soil production rate, consequently greatly altering erosion rates and processes.

At this point, equation (11) is an empiricism derived from the inference that the frequency of contact with the soil-bedrock boundary by potentially disturbing agents (most commonly biota) should decline with increasing soil thickness. Because of the possibly dominant role of biota and the dependency of soil production on weathered state of the bedrock, it may be difficult to develop a more mechanistic expression for equation (11). It nonetheless meets the goals of being a geomorphic transport law: it is process-based and can be parameterized from field measurements and used in geomorphic modeling. The production function may matter more than just enabling prediction of soil depth and limiting erosion rate of exposed bedrock. If soil thickness influences the transport process, for example by influencing the transport coefficient, K, in equation (9) then there may be even stronger coupling between the production function and the rate of landscape erosion.

4.2.4 Landslide transport. Landslides often dominate erosion and therefore strongly influence morphology in steep or mechanically weak terrain. Yet, while excellent work has begun, there exists no geomorphic transport law for landslides. This gap exists because of the inherent difficulties of both documenting landslide processes and in applying geomechanical theories applicable to static conditions to an evolving, spatially and temporally variable material in which instability may be driven by intrinsically stochastic precipitation and earthquakes. Landslide flux volumes define power law [e.g. *Kelsey* et al., 1995] or fractal distributions [e.g. *Hovius*, et al., 1997], and hillslopes dominated by landslides tend to show narrowed probability distribution functions of slopes toward threshold values [e.g. *Strahler*, 1950; *Burbank* et al., 1996]. Such observations have served to motivate and guide numerical models [e.g. *Hergarten* and *Neugebauer*, 1998; *Densmore* et al., 1998].

Here we discuss briefly two models that have attempted to translate empirically supported landslide analyses into transport laws. *Schmidt* and *Montgomery* [1995] fit the Culmann one-dimensional limit equilibrium slope stability model to hillslopes with local relief of less than 5 m to greater than 1000 m. This fit suggested that hillslope-scale strength properties (and landslides) may define the relief of hillslopes for a given mean slope. The stability equation can be written as

$$H_c = \left(\frac{4C}{\gamma}\right)\left(\frac{\sin\beta\cos\beta}{1-\cos(\beta-\phi)}\right) \tag{12}$$

where H_c is the hillslope height, C is cohesion, γ is unit weight, β is the average hillslope gradient, and ϕ is the internal friction angle. The Culmann method assumes a planar failure, apparently unlike those analyzed by Schmidt and Montgomery, so the application of this model to this case may be limited. *Densmore* et al. [1998] built upon this observation to construct a set of rules using equation (12) to model the role of bedrock landsliding in evolution of mountains. They correctly note that modeling landslide transport must address several issues: where and when landslides will happen, how big they will be, and where the landslide material comes to rest once set in motion. They cast equation (12) as a probabilistic function of actual relief divided by a critical value which depends on the time since the last landslide and then proposed a set of rules for the size and runout fate of the mobilized sediment.

Tucker and *Bras* [1998] appear to have been the first to explore how topographically driven subsurface flows may influence landslide erosion and resulting landscape morphology. They used the model proposed by *Dietrich* et al. [1992] and *Montgomery* and *Dietrich* [1994], which can be written to specify the threshold drainage area, A, per unit contour length (or cell size, b) at which failure occurs

$$\frac{A}{b} = \left(\frac{T}{q}\right)\left(\frac{\rho_s}{\rho_w}\right)\left(1-\frac{\tan\theta}{\tan\phi}\right)\sin\theta \tag{13a}$$

in which T is transmissivity (the vertical integration of the saturated conductivity), q is the effective precipitation, (ρ_s/ρ_w) is the ratio of soil to water bulk density, θ is the surface slope and ϕ is the angle of internal friction. *Tucker* and *Bras* [1998] modeled erosion by determining unstable material according to equation (13a) coupled with a rule that deposits the eroded material on a lower stable slope. They then explored how this landslide threshold may influence drainage density and the general slope-drainage area relationship of catchments.

Hergarten and *Neugebauer* [1998] explored the statistical properties of topographic evolution associated with erosion by landsliding. They argued that long-term landslide transport depends primarily on slope angle and landslide thickness and proposed the following equation for landslide flux

$$q_s = \alpha\left(H|\nabla z| - B\right) \tag{13b}$$

where α is a rate constant, H is landslide thickness and B is a threshold for displacement. To account for the production of mobile landslide material, *Hergarten* and *Neugebauer* [1998] formulated an expression that depends on probability-driven weathering impacts, stabilization by drying, and energy dissipation by sliding. Equation 13b was used to evolve an arbitrary landscape with channel lowering boundary conditions and the model output exhibited power-law distributions of landslide area and sediment yield. The study does not purport to be applicable to real landscapes; instead, *Hergarten* and *Neugebauer* [1998] interpret their results as evidence for landsliding as self-organized critical process.

Much more needs to be done to develop a geomorphic transport law for landslides. Besides the limit equilibrium approaches employed in equations (12) and (13), the continuum model approaches mentioned in the non-linear transport discussion above may have application to some kinds of landslides. Both equations (12) and (13) can be parameterized from field data. Considerable effort, however, is needed before independently derived parameter estimates can be used in landslide model applications. Currently, landslide models rely on empirical fitting to obtain realistic results. Strength and hydrologic properties of landscapes are highly variable, making it very difficult to compare models with real topography. Little is known about what controls landslide size, frequency of failure at a site, or runout fate, although some progress has been made for debris flow runout [e.g. *Hungr*, 1995; *Iverson*, 1997].

4.2.5 Horton overland flow erosion. Surface wash, induced by overland flow when the rainfall exceeds the infiltration capacity, contributes to erosion where vegetation is sparse and hillslope materials are inherently impermeable. Despite the long recognition of the role of surface wash in landscape evolution and the many mathematical expressions proposed to represent it [e.g. *Gilbert*, 1877; *Horton*, 1945; *Ahnert*, 1967; *Kirkby*, 1971; *Smith* and *Bretherton*, 1972; *Willgoose* et al., 1991a; *Howard*, 1994; *Smith* et al., 1997a,b; *Tucker* and *Bras*, 2000], at present a geomorphic transport law for this process has not been quantified from field measurements. Here we include in the term surface wash the differing effects of sheetwash and rill concentrated erosion. As in the landslide case, surface wash is understandable mechanistically at individual, plot level investigations, but becomes difficult to characterize for full hillslopes or landscapes [*Dunne*, and *Aubry*, 1986]. This difficulty arises because: 1) wash erosion is driven by stochastic rainfall events, the duration and magnitude of which strongly influence the storm-scale travel distance of sediment; 2) the important influence of material properties which are highly variable including infiltration capacity, stone armoring, vegetation coverage and root strength, and cohesive soils [e.g. *Abrahams*, 1994]; 3) the difficulty of obtaining spatially varying rate measurements applicable to geomorphic time scales, and 4) the effects of fine-scale topography on sediment transport and deposition. Furthermore, it may be that surface wash is rarely transport limited (i.e. a supply of loose sediment in excess of capacity). Instead, as *Gilbert* [1877], *Kirkby* [1971], *Howard* [1994,1997], *Tucker* and *Bras* [2000] and many others have noted, surface wash may almost always be partially or fully detachment-limited.

Although there has been considerable effort in the practical realm of managing soil erosion problems associated with landuse to develop predictive models [e.g. *Bryan*, 1990] these models tend to be either too empirical or over parameterized to be useful for geomorphic modeling. The field study by *Evans* et al. [2000] reports an effort to parameterize an overland flow and surface wash model for the purpose of making landform evolution predictions of a waste rock dump. They fit a model that requires at least six parameters to be calibrated, some of which they were not able to determine from their experiments alone (for example, they could not detect a dependency on surface slope). *Evans* et al. [2000] did not show whether the calibrated model could predict observed landforms in eroded waste dumps.

Generally, numerical models have treated surface wash and channel fluvial transport as a single transport law of the form

$$q_s = k(\tau - \tau_c)^n \tag{14}$$

in which q_s is the volumetric sediment transport rate per unit width, k and n are parameters, τ is the boundary shear stress and τ_c is the critical boundary shear stress required to initiate sediment motion. While flume studies of bedload transport support equation (14), such idealized relationships for surface wash may hold only on surfaces of abundant sand. Commonly, equation (14) is simplified to

$$q_s = dA^m S^n \tag{15}$$

in which S is the local slope, A is the drainage area and d, m and n are fitted parameters. While equation (15) is a rational simplification of the linked runoff-erosion process, we can find no empirical basis for it based on field studies that are relevant to geomorphic space and time scales. Prosser and *Rustomji* [2000], however, do provide a summary of

experimental results for sediment rich systems showing support for the form of equation (15). They conclude that best experimental evidence for flows at sediment transport capacity is that $1.0 \leq m \leq 1.8$ and $0.9 \leq n \leq 1.8$, with the best single combination of values being $m = n = 1.4$. Use of such parameters in landscape models must be done with the caution cited above about the complexities created by stochastic rainfall, the influence of plants, soil strength and other material properties, and the effects of transient sediment storage associated with these effects and fine-scale topography.

4.2.6 River channel sediment transport and incision. River incision drives landscape evolution, and, through numerous studies in recent years, there has been a rapidly growing understanding of the how to build geomorphic transport laws to represent the various processes controlling incision. Here we divide these studies into three kinds of processes and review them separately: alluvial bed incision, bedrock incision by fluvial processes, and bedrock incision by debris flows. In all three cases, it has been proposed that the signature of the process is expressed in how the local channel slope varies downstream with increasing drainage area [e.g. *Willgoose* et al., 1991c, *Seidl* and *Dietrich*, 1992; *Snyder* et al., 2000; *Stock* and *Dietrich*, in press]. We conclude this section making some cautionary comments about the usefulness of power-law plots of slope-area relationships and by pointing to processes that need further study.

4.2.6.1 Alluvial bed incision. Of all the transport processes described here, alluvial sediment transport at the river reach scale is by far the best understood, having both strong mechanistic theory and experimental observations from the field and laboratory [e.g. *Parker*, in press]. Despite this knowledge, gaps are significant when such theory is applied to entire river networks in an evolving landscape, even if the rivers are everywhere covered with sediment. Suspended load theory, based on a unlimited sediment source from the bed, is useful in predicting suspended bed material load transport in lowland, sand bedded alluvial rivers, but has little bearing on uplands rivers in which suspended load is set by stochastic introduction from hillslopes. Bedload transport theory requires knowledge or prediction of bed surface grain size distribution (and spatial patchiness), proportion of the bed covered by sediment, amount and size distribution of incoming sediment from hillslopes, and breakdown rates of sediment with transport. Furthermore, channel dimensions influence shear stress values but these dimensions must be assumed in model applications, because no experimentally supported theory exists for the downstream varying size of river channels in evolving landscapes. Transient sediment storage in fans, bars, bed and floodplains strongly dampens the stochastic input of sediment; and sediment pulses tend to rapidly attenuate [*Lisle*, et al., 1997; *Cui* et al., in press]

While studies have begun to explore network-based routing of sediment [e.g. *Benda* and *Dunne*, 1997; *Jacobson* and *Gran*, 1999], we know of no field study over a large system that has documented sediment routing in a manner that could provide significant constraint on sediment transport models needed for landscape evolution modeling. Instead, at present, there has been a tendency to assume that transport can be written as either

$$q_s = k\, q^m S^n \tag{16}$$

or

$$q_s = k\,(\tau_b - \tau_c)^n \tag{17}$$

often simplified to

$$q_s = k\, \tau_b^n \tag{18}$$

In landscape evolution models, equation (16) and (18) are typically further simplified to

$$q_s = k\, A^m\, S^n \tag{19}$$

Here q_s is the sediment transport per unit active bed width, A is drainage area, a proxy for unit water discharge q, S is local slope, τ_b is boundary shear stress for some representative flow, τ_c is the critical boundary shear stress (bed grain size distribution dependent), and m, n and k are fitted parameters. Note that the meanings of k, m and n differ in each equation and in particular, their values in (19) differ from those in the equivalent expression (15) for hillslope transport. While these relationships are rooted in more mechanistic expressions, it has yet to be shown that they apply to river networks, which are downstream transporting systems influenced by sediment supply with self-forming channel dimensions. *Talling* [2000], for example, points to a tendency for the Shields number (dimensionless shear stress) to remain constant and close to critical along river profiles, suggesting the influence of grain size dependent critical shear stress on slope development. Equations (16)-(19) qualify as geomorphic transport laws in that they are based on process mechanics and can be parameterized, but they haven't yet been parameterized at a geomorphic time and spatial scale.

If it is assumed that there is no underlying bedrock (infinitely deep sediment pile), then equation (19) can be substituted into (4) and the long profile of the river predicted. For the case of steady

state incision in which $\partial z/\partial t = 0$, this expression becomes:

$$S = k_s A^{-\alpha} \tag{20}$$

where $k_s = (U/k_g)^{-n}$, in which k_g has transport and geometric coefficients [e.g. *Willgoose*, 1994], and $\alpha = (m\text{-}1)/n$. Using typical values summarized by *Prosser* and *Rutomji* [2000] of $m = n = 1.4$, gives an α value of 0.29, a value similar to that reported by *Hancock* and *Willgoose* [2001] in the simple case of experimental landscape development into non-cohesive sediment (no distinction was made between hillslopes and channels in their analysis). In general, in this transport limited case, if the exponents reported by Prosser and *Rutomji* [2000] are most representative, there should be a tendency for the exponent, α, to be relatively low. *Hancock* and *Willgoose* [2001] report a range of 0.4 to 0.7 for various field sites, but it is not evident that all these values are simply transport-limited; that is, bedrock resistance and limited sediment coverage on the bed may matter.

4.2.6.2 Bedrock incision by fluvial processes. In contrast to the alluvial bed case, much less theory and observations are available on river incision into bedrock. The simplest hypothesis is that river incision, P, is proportional to stream power or boundary shear stress [*Howard* and *Kerby*, 1983; *Seidl* and *Dietrich*, 1992]. This hypothesis now is most commonly written as:

$$P = K_b(QS/w)^n \tag{21}$$

and simplified to

$$P = K_b A^m S^n \tag{22}$$

assuming discharge, Q, and channel bed width, w, vary as power functions of drainage area. The parameter K_b is hypothesized to reflect influences of rock type on erosion rate [e.g. *Stock* and *Montgomery*, 1999]. Here P is analogous to the production term in equation (6), and represents the lowering rate of bedrock.

The few studies to date that have attempted to test equation (22) have used long-term measures of incision rate, measured drainage area and estimated slope to quantify the values of K, m and n [*Seidl* and *Dietrich*, 1994; *Stock* and *Montgomery*, 1999; *Whipple* et al., 2000b; *Snyder* et al., 2000]. While these studies provide some support to equation (22), these and other studies [i.e. *Seidl* and *Dietrich*, 1992; *Slingerland* et al., 1997; *Sklar* and *Dietrich*, 1998; *Weissel* and *Seidl*; 1998; *Hancock* et al., 1998; *Stock* and *Dietrich*, 1999; *Whipple* et al. 2000a] also suggest that equation (22) does not capture many important processes and effects. These include: the role of knickpoint propagation in causing bed lowering; the role of debris flow scour in the steeper, headwater channels; the role of sediment supply in providing tools and in protecting the bed against erosion; the influence of sediment size; the influence of stochastic storm

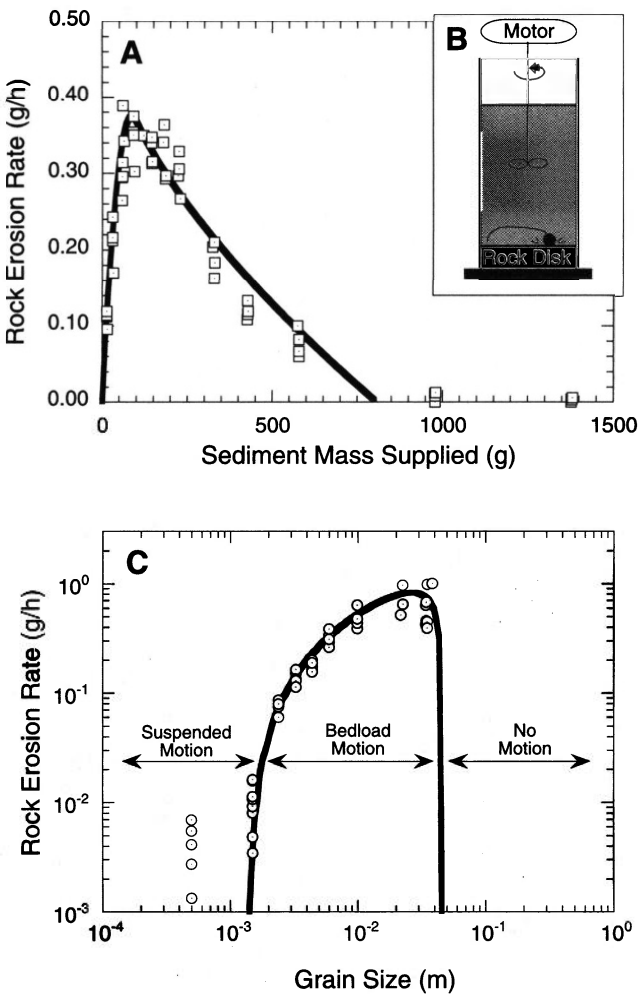

Figure 10. Experimental results demonstrating sediment supply and grain size effects on rates of bedrock wear, shown in grams per hour (g/h). A. Variation in wear rate with sediment supply using the apparatus shown in B. Erosion rate increases with sediment additions until bed armoring by immobile sediment reduces the area of exposed bedrock. Erosion rate declines with further sediment additions until bed is fully buried. C. Wear rate variation with grain size. Wear rate declines with decreasing grain size because of a tendency for small grains to go into suspension and reaches a peak at a grain size that is just above the threshold of motion.

events versus long term average high flows; and the role of rock detachment by plucking, cavitation and abrasion by suspended load, all of which may not scale with stream power.

Sklar and *Dietrich* [1998, 2001] have proposed a mechanistic theory for the influence of sediment supply and grain size on incision rates and conducted experimental studies which strongly support this theory (Figure 10). They assumed that bedrock incision occurs primarily due to abrasion by saltating bedload, and that the rate of rock wear depends linearly on both the flux of particle impact kinetic energy normal to the bed and the fraction of the bed which is not armored by transient deposits of alluvium. In their model, partial alluvial bed cover is assumed to depend on the ratio of coarse sediment supply to bedload transport capacity, and particle impact velocity and impact frequency depend on saltation trajectories, which are parameterized by empirical functions of excess shear stress

As shown in Figure 10A, increasing sediment supply (other conditions being held constant) acts in two essential yet opposing ways: by providing tools for abrasion of exposed bedrock and by limiting the extent of exposure of bedrock in the channel bed. Incision is limited at lower supply rates by a shortage of abrasive tools and at higher supply rates by partial burial of the bedrock substrate beneath transient sediment deposits. As in so many other things, *Gilbert* [p.106, 1877] recognized this possibility when he proposed that the maximum wear rate should occur at an intermediate supply rate.

Sklar and *Dietrich* [1998, 2001] further reasoned that the size distribution of sediment grains supplied to the channel should also influence incision rates because only the coarser fraction is capable of forming an alluvial cover and because the finer fraction is carried in suspension and rarely collides with the bedrock bed. As suggested by Figure 10C, the most efficient abrasive tools are sediments of intermediate size, large enough to travel as bedload but not so large as to be immobile except in the most extreme flows. These data imply that the minimum channel slope that allows a river to incise into bedrock is set primarily by the threshold of motion of the coarse fraction of the sediment load.

The theory proposed by *Sklar* and *Dietrich* [1998, 2001], which combines empirically-based models for bedload sediment transport, particle saltation trajectories and low velocity impact wear, can be expressed as

$$P = \frac{\left(1-\left(u^*/w_f\right)^2\right)^{1.5}}{\varepsilon_v}\left(k_1\frac{q_s}{\left(\tau^*/\tau_c^*-1\right)^{0.5}}-k_2\frac{q_s^2}{D_s^{1.5}\left(\tau^*/\tau_c^*-1\right)^2}\right) \quad (23)$$

where u^* is the shear velocity, w_f is the particle settling velocity, which depends primarily on grain size, D_s, ε_v is a rock strength coefficient that scales with the square of rock tensile strength [*Sklar* and *Dietrich*, 2001], t* is the dimensionless boundary shear stress, τ^*_c is the value of τ^* at the threshold of grain motion, and k_1 and k_2 are fixed-value coefficients determined primarily by sediment density. Equation (23) was derived from consideration of sediment motion and bedrock wear at the temporal and spatial scales of hours and meters. The solid lines in Figure 10 are predictions of the experimental results using parameters determined separately from the experiments. The close match of data and theory adds strong support to equation (23). Applying equation (23) at geomorphic scales requires several key assumptions, including the fraction of the total sediment load in the bedload size class, and the dominant grain diameter and discharge which best represent the net effect of the full distributions of particle size and discharge magnitude and frequency. Unlike simpler incision expressions, such as the stream power model (equation (22)), the bedload saltation-abrasion model explicitly captures only one of several possible bedrock wear mechanisms, although partial bed alluviation should inhibit wear by all incision mechanisms.

Other published models that attempt to account for the role of sediment in bedrock incision include those of *Foley* [1980], which includes the effect of sediment supply in providing tools, and *Beaumont* et al. [1992], which captures the incision inhibiting effect of high sediment supply. The *Beaumont* et al. [1992] model can be written as

$$P = K(q_t - q_s) \quad (24)$$

where q_t is the sediment transport capacity per unit width. *Beaumont* et al. [1992] and others who have used this model in landscape evolution simulations [e.g. *Kooi* and *Beaumont*, 1996; *van der Beek* and *Braun*, 1999] did not use a bedload sediment transport expression, and hence did not capture the potential role of grain size in controlling channel slope and thus landscape relief. With a bedload sediment transport expression for q_t, equation (24) is approximately equivalent to equation (23) for the case of incising rivers carrying near capacity coarse sediment load where underlying bedrock is infrequently exposed.

The substitution of equation (22) into the general mass conservation equation (1) with the assumption of no sediment cover (hence $\partial h/\partial t = 0$), and the assumption of steady state erosion and uplift leads to [e.g. *Howard et al.*, 1994]

$$S = k_b A^{\gamma} \quad (25)$$

in which $k_b = (U/K_b)^{1/n}$ and $\gamma = m/n$. While it is often argued that the ratio m/n should be close to 0.5 [e.g *Whipple* and *Tucker*, 1999; *Rodriquez-Iturbe* and *Rinaldo*, 1997] this is not based on direct determination of m and n from field measurements of erosion rates, stream discharge and topography, but rather from fits to river profiles, which may include a multitude of effects including other processes listed above as well as inaccurate topographic data. Hence, γ, can and does vary widely and will depend on the processes influencing incision. Uplift undoubtedly steepens rivers, and k_b should vary in response [*Snyder* et al. 2000; *Lague*, et al., 2000a,b;], making (25) a potentially useful tool in mapping patterns of uplift [*Kirby* and *Whipple*, 2001], although equation (25) may represent that dependency too simply.

4.2.6.3 Bedrock incision by debris flow. As described in reference to Figure 4, the power law relationship between local channel slope and drainage area tends not to extend above channel slopes of 3 to 10% in non-glaciated steepland valleys in the United States and around the world [e.g, *Stock* and *Dietrich*, in press.]. Above this gradient, where field evidence shows debris flows episodically scour the bedrock [*Stock* and *Dietrich*, 2001, in press], the slope may only slowly change with diminishing drainage area, or more commonly, the area-slope relations are curved, such that with decreasing drainage area the rate of increase of slope progressively declines. This curved relationship suggests a very different topographic dependency for a debris flow incision law than for fluvial. Inclusion of this curved region with the fluvial reaches will tend to systematically reduce the value of γ, underestimating the correct value for the fluvial reach.

The distinction between fluvial and debris flow valley incision is not simply an academic debate about a small portion of the landscape. Valleys with curved area-slope plots, largely above 10% slope, can be both extensive by length (>80% of large steepland basins) and comprise large fractions of mainstem valley relief (25-100%) [*Stock* and *Dietrich*, in press]. As a consequence, most hillslopes in nonglaciated steeplands are bounded by valleys that are carved by debris flows rather than by rivers. Projection of the stream power law to valley heads may result in massive over-predictions of valley slope, and hence relief [*Sklar* and *Dietrich*, 1998; *Stock* and *Dietrich*, in press]. At present there is no geomorphic transport law for debris flows to assess these important issues.

4.2.6.4 Inferring process laws from slope-area data. Some caution is appropriate when making process-based interpretations of slope-area plots. Channels cutting through alluvium, through various bedrock types, under a wide range of uplift and by a variety of processes may all produce similar slope- area relationships. Linear plots in log-log space do not indicate the arrival of a steady state condition. *Sklar* and *Dietrich* [1998] showed that well developed power law slope-area relationships exist for profiles that are clearly still evolving. Furthermore, *Schorghofer* and *Rothman* [2001] demonstrate that as long as flow paths tend to go downhill, there will be a statistical tendency for a slope-area relationship to develop, even in a random topography. The role of sediment supply, grain size, and knickpoint propagation, which most likely are of great importance in channel incision, are not simply captured by the power law expressions (23) and yet may still lead to strong slope-area relationships [e.g. *Sklar* and *Dietrich*, 1998]. Thus slope area scaling, particularly of profiles assumed to be in steady state, is likely to be of limited diagnostic value in testing competing models for river incision into bedrock.

Quantitative small-scale physical modeling experiments may prove to be quite useful in exploring some of these matters. *Lague* et al. [2000a] have conducted experiments on a 20 by 30 cm box of loess (clay and silt) that can be uplifted at a prescribed rate and rained upon. They found a slope-area power law relationship with an exponent of -0.11 on channels all steeper than 10%. The exponent was independent of uplift rate, but the intercept (as mentioned above) varied linearly with uplift rate, as did the resulting average relief. *Lague* et al. [2000] say little about the actual erosion processes, in contrast to *Hancock* and *Willgoose* [2001], who, using a 1.5m by 1.5 m box, give detailed description and emphasize the role of propagating knickpoints in causing channel incision. *Hasbargen* and *Paola* [2000] also note the importance of spontaneously emerging knickpoints in driving channel incision and ridge migration in an experimental landscape at long-term steady state. See also Hasbargen and Paola in this volume.

4.2.7 Glacial erosion. Where glaciers flow, they may dominate all erosion processes, broadening, and locally overdeepening valleys. Quantitative modeling of glacial incision and landscape evolution has begun [e.g. *Harbor*, 1992; *Braun* et al., 1999; *MacGregor* et al., 2000; *Merrand* and *Hallet*, 2000]. At present no erosion rate data are available to guide or test a geomorphic transport law. Instead, these models have used the hypothesis proposed by *Hallet* [1989] that erosion rate, E, depends on basal ice velocity, U_b

$$E = cU_b \tag{26}$$

MacGregor et al. [2000] and *Braun* et al. [1999] describe an approach for estimating U_b, which involves a number of assumptions about ice rheology and form resistance. Such

models based on equation (26) have been useful in offering explanation of glacial features such as "U" shaped valleys [*Harbor*, 1992] and valley profile overdeepening. Other processes, such as stress release and consequent rock avalanching that may further shape the landscape have yet to be included in these models.

Equation (26) may be sufficient to be a geomorphic transport law, but at present it remains unparameterized with field observations of rates of processes.

5. MODELING WITH GEOMORPHIC TRANSPORT LAWS

The ultimate goal of the development and quantification of geomorphic transport laws is their use in numerical models to explore controls on the form and evolution of landscapes. Here we comment briefly on the use of geomorphic transport laws in modeling applications and draw a distinction between their use in hypothetical landscapes, in which the landscape is created entirely by the numerical model, and in real landscapes, in which the initial landscape is real and is subsequently modified by application of geomorphic transport laws and boundary conditions.

5.1 Hypothetical Landscapes

The seminal studies by *Culling* [1960, 1963, 1965], *Kirkby* [1971], *Carson* and Kirkby [1972], *Smith* and *Bretherton* [1972], and *Ahnert* [1976] established the approach of using geomorphic transport laws for different transport and erosion mechanisms to explore controls on landforms and their evolution. The widespread use of the exponents m on drainage area and n on slope (e.g. equations 19 and 22) was first proposed by *Kirkby* [1971]. The papers by *Koons* [1989] and *Willgoose* et al.[1991] marked the next step, in which tectonics [*Koons*] and whole drainage basin modeling [*Willgoose*] were analyzed. These two papers also represent a branching in approach, in that there has been a tendency for papers concerned with larger-scale linkages with tectonics to use large grids, whereas the drainage basin models have tended to use finer grids to capture finer scale features such as individual hillslope shape, drainage density, and channel head locations. In both of these cases, the approach has been to create hypothetical landscapes and compare, using various measures, hypothetical landscapes with real ones. A third path, and one that has attracted broad interest, is the use of erosion rules in numerical models to explore tendencies in self-organization and scaling [e.g. *Rodriguez-Iturbe* and *Rinaldo*, 1997; *Veneziano* and *Niemann*, 2000a,b]. Because this path is purposely not concerned with the specifics of geomorphic transport laws, we will not discuss it further here and refer the reader to the book by *Rodriguez-Iturbe* and *Rinaldo* [1997].

As discussed above, the reliance of large grid models on geomorphic transport laws, which are motivated by and scaled to real processes, is problematic. At present these grids must be large in order to permit computations that explore large space and time domains to reach completion in reasonable computational time. While care has been taken to discuss issues of scaling transport laws, especially the diffusivity term in equation (9), since the *Koons* [1989] study [e.g. van der *Beek* and *Braun*, 1998; *Hurtez* et al., 1999], it seems less appreciated that there simply are not hillslopes and valleys in real landscapes that can be portrayed by 100 to 1000 km scale grids [*Dietrich* and *Montgomery*, 1998]. Models employing these scales implicitly assume that the net effect of the combined and interacting effects of hillslope sediment transport and erosion by the finest scale channel networks act in a collective way that can be captured by selected transport laws. As mentioned above, such integral relationships, as *Anderson* [1994] and *Howard* et al. [1994] have proposed, may exist and be quite useful in exploring large-scale linkages of tectonics and erosion, but they have yet to be demonstrated. On local scales, some researchers have attempted to apply the geomorphic transport laws at appropriate scales while exploring linkages to tectonics [e.g. *Arrowsmith* et al., 1996].

Drainage basin models have attempted to use appropriately scaled transport laws and grid sizes to explore a wide range of morphologic and evolutionary behavior [e.g. *Willgoose* et al., 1991a,b,c,d; *Willgoose*, 1994a,b; *Willgoose* and *Hancock*, 1998; *Moglen* and *Bras*, 1994; *Howard*, 1994, 1997; *Tucker* and *Slingerland*, 1997; *Tucker* and *Bras*, 1998, 2000]. Some primary findings of these studies include: 1) general geomorphic laws give slope-area relationships like those found in real landscapes for constant, varying and zero uplift rates; inflection points in slope-area trends may give clues about process dominance; 2) ridge and valley topography can form without the influence of a threshold slope or a critical shear stress; 3) under constant uplift, steady state topography may form after the total erosion exceeds the steady state relief by a factor of three or more; 4) detachment-limited and transport limited landscapes evolve distinctly different topography; 5) landslide dominated hillslope erosion leads to reduced drainage density and higher relief; 6) cyclic climatic change may be asymmetric in geomorphic response, in which channel expansion is rapid, but retraction is slow; and 7) with increasing climatic variability, the combined effects of stochastic rainfall and threshold and/or nonlinear sediment transport causes erosion rates and drainage density to increase but relief to decrease.

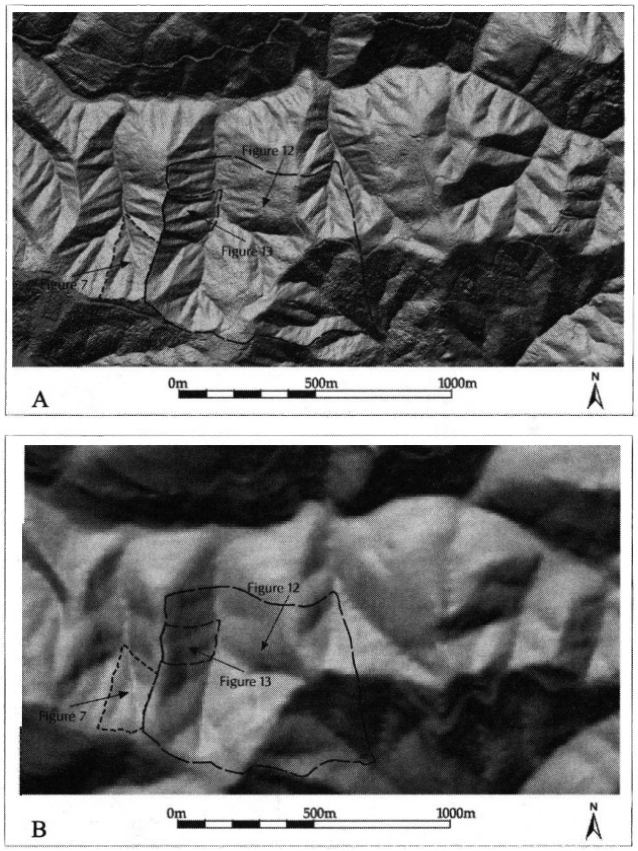

Figure 11. Shaded relief maps of a portion of the Oregon Coast Range near Coos Bay. A. Map based on 2.5-m data density derived from airborne laser swath mapping. B. Map of identical area as A., derived from 10-m grids created from digitized 40-foot contour lines provided by the U.S. Geological Survey. Also shown are the boundaries of the landscape used in the numerical model (and reported in Figure 12), location of a detailed area shown in Figure 13, and the area used to document curvature-slope relationships (Figure 7).

To a large degree, the tectonically linked models and the drainage basin models have relied upon the notion that they are based on geomorphic transport laws, rather than a set of ad-hoc rules, to justify deriving mechanistic insights about landscape dynamics and form from numerical experiments. Neither approach has generally made detailed comparisons with real landscapes using locally calibrated model parameters.

In all landscape evolution models and in nearly all digital elevation models of real landscapes, channel dimensions are neither predicted nor observed. In landscape models, channel dimensions are either imposed empirically or ignored. In digital terrain models, cells are assigned a category (hillslope or channel) based on slope criteria, typically a threshold slope. This is a significant theoretical and observational gap.

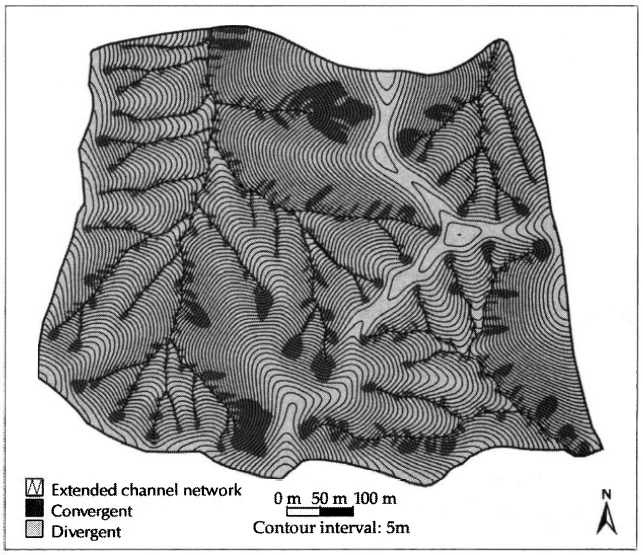

Figure 12. Maps of total topographic convergence or divergence after one million years of constant channel incision. Channel network used is an extension of the current channels to account for effects of debris flow scour. Time step was 1 year. Field calibrated parameters used in equation (10) are K_{nl} = 0.0032 m^2/yr and S_c = 1.25 [*Roering et al.*, 1999]. To convert to a mass flux we multiplied the right hand size of (10) by the soil bulk density (ρ_s = 800 kg/cm^3). Field calibrated parameters used in equation (11) are ε_0 = 0.000268 m/yr, α = 0.03 (1/m) [*Heimsath et al.*,2001]. In this case, to convert to mass flux, we multiplied the left hand side of equation (11) by the rock bulk density (ρ_r = 2270 kg/m^3) and the right hand side by the soil bulk density. Note the convergent areas above the channel head and upslope of irregularities in channel paths.

5.2 Real Landscapes

Numerical landscape evolution models commonly assume transport laws and boundary conditions and then, for an arbitrary initial topography, solve the conservation of mass equations for time steps over some selected period. Resulting topography is then compared in various ways with a proposed analog real topography, as discussed above. An alternative approach, using high-resolution real initial topography, may be useful in building insight about the underlying mechanisms controlling landscape morphology. In this case, for example, a limited set of field parameterized transport laws, perhaps less than that necessary to model the landscape completely, could be used to see how the transport laws modify the existing topography from its current state. Changes in the landscape or, alternatively, persistence of landforms from the initial state may then shed light on the role of these processes in shaping the landscape, and point to the role of other processes not modeled. Such predictions

to some degree are expected to fail, but in the failure some insight may be gained about the underlying mechanisms.

Here we briefly summarize the results one such numerical experiment to illustrate this idea. Figure 11 shows Sullivan Creek, a basin in the Oregon Coast Range, where *Heimsath* et al. [2001] and *Roering* et al. [1999] have calibrated soil production (equation 11) and non-linear soil transport (equation 10) laws, and long-term rates of channel incision have been estimated [*Reneau* and *Dietrich*, 1991; *Heimsath* et al., 2001]. Figures 11A and 11B illuminate the sharp contrast between topography derived from airborne laser swath mapping (which gave an average bare ground data density of 2.5 m) which was used in the modeling, and 10 m gridded data (from the U.S Geological Survey contour maps. A simple hypothesis was tested: if the landscape is approximately in steady state and we have accurately quantified the topography, transport laws and boundary conditions, then forward numerical modeling of the erosion of the real topography should produce only minor morphologic change. To perform this test, we gridded a portion of the landscape (Figure 11A and Figure 12), lowered the channel network at the estimated long-term erosion rate (100m/My), and applied the locally calibrated non-linear transport law and the soil production function.

Figure 12 shows the pattern of steady-state convergent and divergent topography after 1 million years. The convergent zones above the channel heads were a surprise. Since the work by *Smith* and *Bretherton* [1972], it has been generally assumed that convergent topography results from processes for which transport capacity increases with drainage area, such as overland flow erosion or storm-driven shallow landsliding. In our model, channel incision forces the contours to curve around the channel boundary. If transport varied linearly with slope, the curvature of contours would be compensated by strong profile curvature, such that the net total curvature is divergent. In the non-linear case, however, as the profile steepens up into the strongly non-linear range of behavior, transport increases so much that it can accommodate the upslope convergent sediment flux. Hence, to our surprise, the non-linear transport law can support steady state convergent topography. Figure 12 also shows that there are numerous small hollows that line up along the steep slopes bordering the channels. Each hollow is associated with small changes in direction of the local channel boundary, which create a local corner in the topography. This corner creates a slight convergence in topography. If the adjacent slopes are sufficiently steep that the transport there is strongly non-linear, the effects of this slight convergence may propagate well upslope, creating local hollows. This suggests that some of the present hollows may have originated from planform boundary irregularities in the channel network that were propagated up the hillslope due to the non-linear sediment transport process. We expect, however, that the topographic convergence would also lead to shallow subsurface flow convergence and increased probability of landsliding.

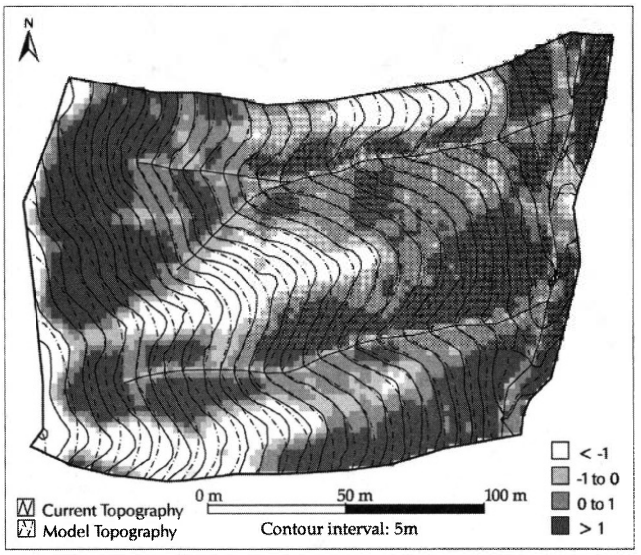

Figure 13. Topographic change for a portion of the study area, showing the initial and final (after one million years) contours and a map of the net difference in elevation (after adding back the total channel lowering to the one million year contour lines). Generally the sharp irregular ridges are lowered and broadened and the broad concavities bordering the channel are narrowed and the elevation increased.

Figure 13 shows a small portion of the map area, where within about 50,000 model years (5 m of total channel lowering) steady state was reached, hence this is an area that was initially relatively close to the "unchanging" condition expected for a perfect match of boundary conditions and geomorphic transport laws. Figure 13 compares the initial and final contour lines and maps the net elevation change between the two. Here and across the study area generally we see, relative to the modeled steady state topography, that the initial contour lines on the narrow ridges tend to be more sharply curved and the contours along the valley axes tend to be more broadly concave (downslope). There are also, however, many contours lines that differ little in detail between the initial and final values.

These observations give support to the non-linear transport law, through the relatively rapid arrival at steady state and the similarities of many contours, but also indicate important differences. Some of these differences arise from the absence in this model of shallow landsliding and debris flow scour which periodically extend the channel network farther upslope and which maintain the numerous tributary

hollows. These processes probably contribute to the formation of the more extensive convergent areas overall, although how this contributes to the broad convergent areas along the channels is not clear. *Heimsath* et al. [2001] noted that the bedrock along the narrow ridges was less weathered and, under a given soil depth, produced less soil. This effect may, by some process not documented, influence the interaction with transport processes in a way that contributes to stronger ridge curvature than average. Differences in modeled and observed topography also may result from the natural landscape having nonuniform channel incision and variable bedrock resistance.

During the model run, hillslopes remained soil mantled and soil thickness eventually varied slightly with slope, as expected for steady state conditions if soil production is normal to the bedrock surface [e.g. *Heimsath* et al., 2001]. If the soil production rate had been lower or the transport rate higher, local areas of bedrock would have emerged. The absence of extensive bedrock exposure in the model adds some support to the applicability of the soil production function here.

6. NEEDS AND OPPORTUNITIES

Given the fundamental role geomorphic transport laws play in ongoing efforts to understand the form and evolution of landscapes, it is surprising that there have been so few studies done to develop and verify the form of these equations. While more work needs to be done on all processes, large opportunities exist in some areas. We lack either the geomorphic transport law or any field parameterization of proposed laws for many processes, including landslides, solution effects, Horton overland flow surface wash and rainsplash, debris flows, seepage erosion, glacial wear, periglacial hillslope transport, and processes that erode bedrock dominated landscapes in general. There is also a need to explore how to scale up sets of local processes [e.g. *Anderson*, 1994; *Davy* and *Crave*, 2000] so that numerical modeling conducted with catchment scale grid cells can move from apparent realism to coarse realism.

At present there is no theory for and very limited quantitative observation of channel cross-sectional area through river networks [see *Montgomery* and *Gran*, 2001 for data on bedrock channels]. Because channel width can change dramatically and directly influence hypothesized channel incision rates, this gap in theory is significant for channel incision models. The initial work by *Rinaldo* et al. [1995] and *Tucker* and *Bras* [2000] on the role of rainfall variability on erosion and sediment transport are first steps away from steady state assumptions. The influence of bedrock variability on landscape evolution, form and sediment discharge variability has received little field or modeling study. All measures of local properties of the landscapes show considerable variability, which tend, for example, to obscure trends in plots of local slope against drainage area.

With new tools available to obtain high-resolution topographic data and determine rates of processes, the opportunity is now upon us to quantify the processes that are responsible for shaping the earth's surface.

7. CONCLUSIONS

The notion of geomorphic transport laws is based on the following premise: mathematical expressions derived from mechanics or physical principles can be made sufficiently simple that they can be parameterized through field observations and physical experiments, and, when used in landscape evolution models, enable reliable inferences about causality to be made. It is hypothesized that such laws can be accurately expressed on an intermediate scale that is large enough to be applicable to landscape modeling, but also small enough that the laws represent transport mechanisms with acceptable fidelity. Application at this intermediate scale may require a relaxation of strict, fully defined mechanical modeling, but, unlike rules-based models, maintains a direct and explicit connection to the underlying physics. Geomorphic transport laws are a compromise, then, between fully physics-based and completely rules-based transport equations, but this compromise may strike the balance that enables us to make discoveries through numerical modeling as well as to be able to reject theories that don't fit the data.

Geomorphic transport laws are appropriate, we propose, for modeling that has as a goal of exploring what we have called essential realism. In this view, measures can be defined that distinguish different landscapes and test appropriate transport laws. Exact prediction of specific features at specific locations and times is not the intention of this approach, although some limited testing can be done using high resolution topography. Those concerned with the general spatial structure of landscapes, what we have called statistical realism, may still find parameterizable geomorphic transport laws too specific. Instead they will focus on the general form of the equations and how their linkage in modeling of evolving landscapes can give rise to self-organized patterns. We caution against process interpretation of large scale models of landscape evolution in which geomorphic transport laws have been applied at scales at which the processes they represent simply don't occur. Models of this kind may give rise to an apparent realism only. Society often demands knowledge at specific sites for specific time periods in which a detailed realism in modeling may be

necessary. Geomorphic transport laws may provide a kind of reference state for such applications, but new understanding is needed that will permit short term prediction of such things as sediment discharge to rivers, the routing of sediment through networks and corresponding morphologic response of the channel.

At present, just a handful of studies have provided evidence for and parameterization of some of the key transport and erosion processes that shape the earth. There is evidence for linear and nonlinear geomorphic transport laws on hillslopes. Soil production from bedrock appears to vary with thickness of the overlying soil. River incision into bedrock varies with shear stress, stream power, sediment supply, grain size, and bedrock strength, but also is influenced by knickpoint propagation and, in steep reaches, by periodic debris flow. No geomorphic transport laws have been parameterized from field observations for many important processes including surface wash, landsliding and glacial scour. Thanks to new high-resolution topography and dating tools, this knowledge gap should narrow.

The advent of high-resolution topographic data sets offers the possibility of performing numerical modeling experiments using real topography as the initial condition. Preliminary trials with this approach show that it can be used to test the applicability of geomorphic transport laws and their underlying assumptions. This approach also may reveal previously unrecognized mechanisms underlying morphodyanmics. Such topographic detail, furthermore, is essential to conducting studies of what we have called detailed realism.

The influence of variability in precipitation and material properties, as well as the need to be able to model large scale linkages between tectonics, climate and erosion present significant challenges to the notion of geomorphic transport laws. It remains to be seen whether these complexities can be treated with sufficient simplicity such that transport laws and model outcomes can be tested with observations from real landscapes.

Acknowledgements. Peter Wilcock, Mike Kirkby, Greg Tucker and an anonymous reviewer provided valuable comments on an earlier draft of this paper. We have had many conversations with colleagues about transport laws, and David Furbish, Alan Howard, Gary Parker, John Chappell, Andrea Rinaldo and Kelin Whipple were especially helpful. We acknowledge financial support from NASA, NSF grants (including support from the National Center for Earth-Surface Dynamics), and a gift from Stillwater Sciences.

REFERENCES

Abrahams, A.D., A.D. Howard, and A.J. Parsons, Rock-mantled slopes, in *Geomorphology of Desert Environments*, edited by A. Abrahams, and A.J. Parsons, pp. 172-212, Chapman & Hall, London, 1994.

Ahnert, F., The role of the equilibrium concept in the interpretation of landforms of fluvial erosion and deposition, in *L'evolution des Versants*, edited by P. Macar, pp. 23-41, University of Liege, Liege, 1967.

Ahnert, F., Brief description of a comprehensive three-dimensional process-response model of landform devel opment, *Zeit. für Geomorph., 25*, 29-49, 1976.

Ahnert, F., Modelling landform change, in *Modelling Geomorphological Systems*, edited by M.G. Anderson, pp.375-400., J. Wiley, 1988.

Anderson, R.S., Evolution of the Santa Cruz Mountains, California, through tectonic growth and geomorphic decay, *J. Geophys. Res., 99*, 20,161-20,174, 1994.

Anderson, R.S., and N.F. Humphrey, Interaction of weathering and transport processes in the evolution of arid landscapes, in *Quantitative Dynamic Stratigraphy*, edited by T.A. Cross, pp. 349-361, Prentice Hall, Englewood Cliffs, NJ, 1989.

Anderson, R.S., Modeling the tor-dotted crests, bedrock edges, and parabolic profiles of high alpine surfaces of the Wind River Range, Wyoming, *Geomorphology, 46*, 35-58, 2002.

Andrews, D.J., and R.C. Bucknam, Fitting degradation of shoreline scarps by a nonlinear diffusion model, *J. Geophys. Res., 92*, 12,857-12,867, 1987.

Arrowsmith, J.R., D.D. Pollard, and D.D. Rhodes, Hillslope development in areas of active tectonics, *J. Geophys. Res., 101*, 6255-6275, 1996.

Avouac, J.-P., and G. Peltzer, Active tectonics in southern Xinjiang, China; analysis of terrace riser and normal fault scarp degradation along the Hotan-Qira fault system, *J. Geophys. Res., 98*, 21,773-21,807, 1993.

Bates, R.L., and J.A. Jackson, *Dictionary of Geological Terms*, 571 p., Anchor Books, New York, 1984.

Beaumont, C.P., P. Fullsack, and J. Hamilton, Erosional control of active compressional orogens, in *Thrust Tectonics*, edited by K.R. McClay, pp. 1-18, Chapman and Hall, New York, 1992.

Benda, L., and T. Dunne, Stochastic forcing of sediment supply to channel networks from landsliding and debris flow, *Water Resourc. Res., 33*, 2849-2863, 1997.

Braun, J., D. Zwartz, and J. Tomkin, A new surface processes model combining glacial and fluvial erosion, *Annals of Glaciology*, p. 282-290, 1999.

Brozovic, N., D. Burbank, and A. Meigs, Climatic limits on land scape development in the Northwestern Himalaya, *Science, 276*, 571-574, 1997.

Bryan, R.B., Soil erosion: Experiments and models, *Catena Suppl. 17*, pp. 208, 1990.

Burbank, D.W., J. Leland, E. Fielding, R.S. Anderson, N. Brozovic, M.R. Reid, and C. Duncan, Bedrock incision, rock uplift and thresh old hillsslopes in the Northwestern Himalayas, *Nature, 379*, 505-510, 1996.

Carson, M.A., and M.J. Kirkby, *Hillslope form and process*, 475 pp., Cambridge University Press, Cambridge, 1972.

Chase, C., Fluvial landsculpting and the fractal dimension of topography, *Geomorphology, 5*, 39-57, 1992.

Cox, N.J., On the relationship between bedrock lowering and regolith thickness, *Ear. Surf. Proc. and Landforms*, *5*, 271-274, 1980.

Cui, Y., Parker, G., Lisle, T.E., Gott, J., Hansler, M.E., Pizzuto, J.E., Allemdinger, N.E., and Reed, J.M., Sediment pulses in mountain rivers, I. Experiments, *Water Resourc. Res.,* in press.

Culling, W.E.H., Analytical theory of erosion, *J. Geol.*, *68*, 336-344, 1960.

Culling, W.E.H., Soil creep and the development of hillside slopes, *J. Geol.*, *71*, 127-161, 1963.

Culling, W.E.H., Theory of erosion on soil-covered slopes, *J. Geol.*, *73*, 230-254, 1965.

Davis, W.M., The convex profile of badland divides, *Science*, *20*, 245, 1892.

Davis, W.M., Base-level, grade, and peneplain, *J. Geol.*, 10, 77-111, 1902

Davy, P., and A. Crave, Upscaling local-scale transport processes in large-scale relief dynamics, *Phys. and Chem. of the Earth (A)*, *25*, 533-541, 2000.

Densmore, A.L., M.A. Ellis, and R.S. Anderson, Landsliding and the evolution of normal-fault-bounded mountains, *J. Geophys. Res.*, *103*, 15,203-15,219, 1998.

Densmore, A.L., and N. Hovius, Topographic fingerprints of bedrock landslides, *Geology*, *28*, 371-374, 2000.

Dietrich, W.E., and D.R. Montgomery, Hillslopes, channels, and landscape scale, in *Scale dependence and scale invariance in hydrology*, edited by G. Sposito, pp. 30-60, Cambridge University Press, Cambridge, 1998.

Dietrich, W.E., R. Reiss, M.-L. Hsu, and D.R. Montgomery, A process-based model for colluvial soil depth and shallow landsliding using digital elevation data, *Hydrol. Proc.*, *9*, 383-400, 1995.

Dietrich, W.E., C.J. Wilson, D.R. Montgomery, J. McKean, and R. Bauer, Erosion thresholds and land surface morphology, *Geology*, *20*, 675-679, 1992.

Dunne, T., and B.F. Aubry, Evaluation of Horton's theory of sheet wash and rill erosion on the basis of field experiments, in *Hillslope Processes*, edited by A.D. Abrahams, pp. 31-53, 1986.

Evans, K.G., and G.R. Willgoose, Post-mining landform evolution modelling: 2. Effects of vegetation and surface ripping, *Ear. Surf. Proc. and Landforms*, *25*, 803-823, 2000.

Fernandes, N.F., and W.E. Dietrich, Hillslope evolution by diffusive processes: the timescale for equilibrium adjustments, *Water Resourc. Res.*, *33*, 1307-1318, 1997.

Fleming, R.W., and A.M. Johnson, Rates of seasonal creep of silty clay soil, *Quart. J. Engin. Geol.*, *8*, 1-29, 1975.

Foley, M.G., Bedrock incision by streams, *Geol. Soc. Am. Bull. pt. II*, *91*, 2189-2213, 1980.

Furbish, D.J., and W.E. Dietrich, A physical interpretation of the diffusion-like coefficient in hillslope evolution models based on probability theory, *Geol. Soc. of Am., Abst. Prog., Ann. Mtg.*, 2000.

Gabet, E.J., Gopher bioturbation: Field evidence for non-linear hillslope diffusion, *Ear. Surf. Proc. and Landforms*, *25*, 1419-1428, 2000.

Gilbert, G.K., Report on the Geology of the Henry Mountains (Utah), U.S. Geol. Surv., Washington, D.C., 1877.

Gilbert, G.K., The convexity of hilltops, *J. Geol.*, *17*, 344-350, 1909.

Hack, J.T., Studies of longitudinal stream profiles in Virginia and Maryland, *U. S. Geol. Surv. Prof. Paper 294-B*, 45-97, 1957.

Hallet, B., A theoretical model for glacial abrasion, *J. Glaciol.*, *23*, 39-50, 1979.

Hancock, G., and G. Willgoose, The interaction between hydrology and geomorphology in a landscape simulator experiment, *Hydrol. Proc.*, *15*, 115-133, 2001.

Hancock, G.S., R.S. Andersen, and K.X. Whipple, Beyond power: bedrock river incision process and form, in *Rivers over Rock*, edited by K. Tinkler, and E.E. Wohl, pp. 35-60, Am. Geophys. Un., Washington, D.C., 1998.

Hanks, T.C., and R.E. Wallace, Morphological analysis of the Lake Lahontan shoreline and Beachfront fault scarps, Pershing County, Nevada, *Bull. Seismol. Soc. Am.*, *75*, 835-846, 1985.

Harbor, J.M., Numerical modelling of the development of U-shaped valleys by glacial erosion, *Geol. Soc. Am. Bull.*, *104*, 1364-1375, 1992.

Hasbargen, L.E., and C. Paola, Landscape instability in an experimental drainage basin, *Geology*, *28*, 1067-1070, 2000.

Heimsath, A.M., The soil production function, Ph.D thesis, 324 p., University of California, Berkeley, Calif., 1999.

Heimsath, A.M., J. Chappell, W.E. Dietrich, K. Nishiizumi, and R.C. Finkel, Soil production on a retreating escarpment in southeastern Australia, *Geology*, *28*, 787-790, 2000.

Heimsath, A.M., J. Chappell, W.E. Dietrich, K. Nishiizumi, and R.C. Finkel, Late Quaternary erosion in southeastern Australia: a field example using cosmogenic radionuclides, *Quat. Intl.*, *83-85*, 169-185, 2001.

Heimsath, A.M., W.E. Dietrich, K. Nishiizumi, and R.C. Finkel, The soil production function and landscape equilibrium, *Nature*, *388*, 358-361, 1997.

Heimsath, A.M., W.E. Dietrich, K. Nishiizumi, and R.C. Finkel, Cosmogenic nuclides, topography, and the spatial variation of soil depth, *Geomorphology*, *27*, 151-172, 1999.

Heimsath, A.M., W.E. Dietrich, K. Nishiizumi, and R.C. Finkel, Stochastic processes of soil production and transport: Erosion rates, topographic variation and cosmogenic nuclides in the Oregon Coast Range, *Ear. Surf. Proc. and Landforms*, *26*, 531-552, 2001.

Hergarten, S., and H.J. Neugebauer, Self-organized criticality in a landslide model, *Geophys. Res. Lett.*, *25*, 801-804, 1998.

Horton, R.E., Erosional development of streams and their drainage basins; hydrophysical approach to quantitative morphology, *Bull. Geol. Soc. Am.*, *56*, 275-370, 1945.

Hovius, N., C.P. Stark, and P.A. Allen, Sediment flux from a mountain belt derived by landslide mapping, *Geology*, *25*, 231-234, 1997.

Howard, A.D., A detachment-limited model of drainage basin evolution, *Water Resourc. Res.*, *30*, 2261-2285, 1994.

Howard, A.D., Simulation modeling and statistical classification of escarpment planforms, *Geomorphology*, *12*, 187-214, 1995.

Howard, A.D., Badland morphology and evolution: interpretation using a simulation model, *Ear. Surf. Proc. and Landforms*, *22*, 211-227, 1997.

Howard, A.D., Long profile development of bedrock channels: Interation of weathering, mass wasting, bed erosion, and sedi-

ment transport, in *Rivers over rock: fluvial processes in bedrock channels*, edited by K. Tinkler, and E. Wohl, pp. 297-320, Am. Geophys. Un., 1998.

Howard, A.D., W.E. Dietrich, and M.A. Seidl, Modeling fluvial erosion on regional to continental scales, *J. Geophys. Res., 99*, 13,971-13,986, 1994.

Howard, A.D., and G. Kerby, Channel changes in badlands, *Geol. Soc. Am. Bull., 94*, 739-752, 1983.

Hungr, O., A model for the runout analysis of rapid flow slides, debris flows, and avalanches, *Can. Geotech. J., 32*, 610-623,1995.

Hurtrez, J.-E., F. Lucazeau, J. Lave, and J.-P. Avouac, Investigation of the relationships between basin morphology, tectonic uplift, and denudation from the study of an active fold belt in the Siwalik Hills, central Nepal, *J. Geophys. Res., 104*, 12,779-12,796, 1999.

Ibbitt, R.P., G.R. Willgoose, and M.J. Duncan, Channel network simulation models compared with data from the Ashley River, New Zealand, *Water Resourc. Res., 35*, 3875-3890, 1999.

Ikeda, S. and Parker, G., *River Meandering*, 485 p., Water Resources Monograph, Am. Geophys. Un., Wash. D.C., 1989.

Iverson, R.M., The physics of debris flows, *Rev. Geophys., 35*, 245-296, 1997.

Izumi, H., and G. Parker, Inception of channelization and drainage basin formation: upstream-driven theory, *J. Fluid Mech., 283*, 341-363, 1995.

Jacobson, R.B., and K. Gran, Gravel sediment routing from wide spread, low-intensity landscape disturbance, Current River basin, Missouri, *Ear. Surf. Proc. and Landforms, 24*, 897-917, 1999.

Kelsey, H. M., M. Coghlan, J.C. Pitlick, and D.W. Best, Geomorphic analysis of streamside landslides in the Redwood Creek basin, northwestern California, U. S. Geological Survey Professional Paper 1454, pp. J1-J12, 1995.

Kirby, E., and K. Whipple, Quantifying differential rock-uplift rates via stream profile analysis, *Geology, 29*, 415-418, 2001.

Kirkby, M.J., Hillslope process-response models based on the continuity equation, *Inst. Brit. Geogr., Spec. Publ. 3*, 15-30, 1971.

Kirkby, M.J., Modelling cliff development in South Wales; Savigear re-reviewed, *Zeit. fur Geomorph. 28*, 405-426, 1984.

Kirkby, M.J., A model for the evolution of regolith-mantled slopes, in *Models in Geomorphology*, edited by M.J. Woldenberg, pp. 213-237, Allen & Unwin, 1985.

Kooi, H., and C. Beaumont, Escarpment evolution on high-elevation rifted margins: insights derived from a surface processes model that combines diffusion, advection, and reaction, *J. Geophys. Res., 99*, 12,191-12,209, 1994.

Kooi, H., and C. Beaumont, Large-scale geomorphology; classical concepts reconciled and integrated with contemporary ideas via a surface processes model, *J. Geophys. Res., 101*, 3361-3386, 1996.

Koons, P.O., The topographic evolution of collisional mountain belts; a numerical look at the Southern Alps, New Zealand, *Am. J. Sci., 289*, 1041-1069, 1989.

Lague, D., A. Crave, J. Kermarrec, and P. Davy, Experimental study of the interaction between erosion and uplift, Geosciences, Rennes, France, 2000a.

Lague, D., P. Davy, and A. Crave, Estimating uplift rate and erodibility from the area-slope relationship: Examples from Brittany (France) and numerical modelling, *Phys. and Chem. of the Earth (A), 25*, 543-548, 2000b.

Leopold, L.B., and W.B. Langbein, The concept of entropy in land scape evolution, pp. 20, U. S. Geol. Surv., *Prof. Paper 500-A*, 1962.

Li, Y.K., G.N. Liu, and Z.J. Cui, Glacial valley cross-profile morphology, Tian Shan Mountains, China, *Geomorpology, 38*, 153-166, 2001.

Lisle, T.E., J.E. Pizzuto, H. Ikeda, and F. Iseya, Evolution of a sediment wave in an experimental channel, *Water Resourc. Res., 33*, 1971-1981, 1997.

Loewenherz, D.S., Stability and the initiation of channelized sur face drainage; a reassessment of the short wavelength limit, *J. Geophys. Res., 96*, 8453-8464, 1991.

MacGregor, K.R., R.S. Anderson, S.P. Anderson, and E.D. Waddington, Numerical simulations of glacial-valley longitudinal profile evolution, *Geology, 28*, 1031-1034, 2000.

Mackin, J.H., Concept of the graded river, *Geol. Soc. of Am. Bull., 5*, 463-511, 1948.

Martin, Y., Modelling hillslope evolution: linear and nonlinear transport relations, *Geomorphology, 34*, 1-21, 2000.

Merrand, Y., and B. Hallet, Glacial landscapes evolution in subpolar regions: an integrated field and theoretical study to model the evolution of the Chugach-St. Elias mountains, Southern Alaska, *Eos Trans. AGU*, 81 (48), Fall Meet. Suppl., 2000.

Merritts, D., and K. R. Vincent, Geomorphic response of coastal streams to low, intermediate, and high rates of uplift, Mendocino triple junction region, northern California, *Geol. Soc. Amer. Bull.*, 101, 1373-1388, 1989

McKean, J.A., W.E. Dietrich, R.C. Finkel, J.R. Southon, and M.W. Caffee, Quantification of soil production and downslope creep rates from cosmogenic ^{10}Be accumulations on a hillslope pro file, *Geology, 21*, 343-346, 1993.

Moeyersons, J., and J. de Ploey, Quantitative data on splash erosion, simulated on unvegetated slopes, *Zeit. fur Geomorph, Suppl., 25*, 120-131, 1976.

Moglen, G.E., and R.L. Bras, The effect of spatial heterogeneities on geomorphic expression in a model of basin evolution, *Water Resourc. Res., 31*, 2613-2623, 1995.

Montgomery, D.R., and W.E. Dietrich, Where do channels begin?, *Nature, 336*, 232-234, 1988.

Montgomery, D.R., and W.E. Dietrich, Channel initiation and the problem of landscape scale, *Science, 255*, 826-830, 1992.

Montgomery, D.R., and E. Foufoula-Georgiou, Channel networks sources representation using digital elevation models, *Water Resourc. Res., 29*, 1925-1934, 1993.

Montgomery, D.R., and W.E. Dietrich, A physically based model for the topographic control on shallow landsliding, *Water Resourc. Res., 30*, 1153-1171, 1994.

Montgomery, D.R., and K. Gran, Downstream variation in the width of bedrock channels, *Water Resourc. Res., 37*, 1841-1846, 2001.

Montgomery, D.R., Slope distributions, threshold hillslopes, and steady-state topography, *American Journal of Science, 301*, 432-454, 2001.

Mosley, M.P., Rainsplash and the convexity of badland divides, *Zeit, fur Geomorph., Suppl., 18*, 10-25, 1973.

Nordin, C.F., The sediment loads of rivers, in *Facets of Hydrology II*, edited by J.C. Rodda, pp. 183-204, John Wiley & Sons, Chichester, 1985.

Ohmori, H., Change in the mathematical function type describing the longitudinal profile of a river through an evolutionary process, *J. Geol.*, 99, 97-110, 1991.

Parker, G., Selective sorting and abrasion of river gravel. 1: Theory, *J. Hydraul. Eng., 117*, 131-149, 1991a.

Parker, G., Selective sorting and abrasion of river gravel. 2: Applications, *J. Hydraul. Eng., 117*, 150-171, 1991b.

Parker, G. and Izumi, N., Purely erosional cyclic and solitary steps created by flow over a cohesive bed, *J. Fluid Mech., 419*, 203-238, 2000.

Parker, G, Transport of gravel and sediment mixtures, in American Society of Civil Engineering Manual 54, revised edition, 131p.,in press.

Parker, R.L., *Geophysical inverse theory*, 386 p. pp., Princeton University Press, Princeton, N.J., 1994.

Perg, L., R.S. Anderson, and R. Finkel, Cosmogenic radionuclide constraints on the long-term sediment budget of the Santa Cruz littoral cell, California, USA, *Eos Trans. AGU*, 81, 2000.

Prosser, I.P., and P. Rutomji, Sediment transport capacity relations for overland flow, *Prog. Phys. Geogr., 24*, 179-193, 2000.

Reneau, S.L., Depositional and erosional history of hollows; application to landslide location and frequency, long-term erosion rates, and the effects of climatic change, Ph. D. thesis, 327 p., University of Calif., Berkeley, Calif., 1988.

Reneau, S.L., and W.E. Dietrich, Erosion rates in the Southern Oregon Coast Range: Evidence for an equilibrium between hill slope erosion and sediment yield, *Ear. Surf. Proc. and Landforms, 16*, 307-322, 1991.

Riebe, C.S., J.W. Kirchner, D.E. Granger, and R.C. Finkel, Erosional equilibrium and disequilibrium in the Sierra Nevada, inferred from cosmogenic Al-26 and Be-10 in alluvial sediment, *Geology, 28*, 803-806, 2000.

Rigon, R., A. Rinaldo, I. Rodriguez-Iturbe, R.L. Bras, and E. Ijjasz-Vasquez, Optimal channel networks - a framework for the study of river basin morphology, *Water Resourc. Res., 29*, 1635-1646, 1993.

Rinaldo, A., Dietrich, W. E., Rigon, R., Vogel, G. K., Rodriguez-Iturbe, I.,Geomorphological signatures of climate, Nature, 374, 632-635,1995.

Rodriguez-Iturbe, I., and A. Rinaldo, *Fractal river basins: Chance and self-organization*, 547 pp., Cambridge University Press, New York, 1997.

Roering, J.J., J.W. Kirchner, and W.E. Dietrich, Evidence for nonlinear, diffusive sediment transport on hillslopes and implica-tions for landscape morphology, *Water Resourc. Res., 35*, 853-870, 1999.

Roering, J.J., J.W. Kirchner, and W.E. Dietrich, Hillslope evolution by nonlinear, slope-dependent transport: Steady-state morphology and equilibrium adjustment timescales, *J. Geophys. Res., 106*, 16,499-16,513, 2001a.

Roering, J.J., J.W. Kirchner, L.S. Sklar, and W.E. Dietrich, Hillslope evolution by nonlinear creep and landsliding: An experimental study, *Geology, 29*, 143-146, 2001b.

Schmidt, K.M., and D.R. Montgomery, Limits to relief, *Science, 270*, 617-620, 1995.

Schorghofer, N., and D.H. Rothman, Basins of attraction on random topography, *Phys. Rev. E, 6302*, U132-U137, 2001.

Seidl, M.A., and W.E. Dietrich, The problem of channel erosion into bedrock, *Catena Suppl., 23*, 101-124, 1992.

Seidl, M.A., W.E. Dietrich, and J.W. Kirchner, Longitudinal profile development into bedrock; an analysis of Hawaiian channels, *J. Geol., 102*, 457-474, 1994.

Selby, M.J., *Hillslope materials and processes*, 451 pp., Oxford Univ. Press, Oxford, 1993.

Shulits, S., Rational equation of river-bed profile, *Trans., Am. Geophys. Un., 22*, 622-630, 1941.

Sklar, L., and W.E. Dietrich, River longitudinal profiles and bedrock incision models: Stream power and the influence of sediment supply, in *Rivers over rock: fluvial processes in bedrock channels*, edited by K. Tinkler, and E. Wohl, pp. 237-260, Am. Geophys. Un., Wash., D.C., 1998.

Sklar, L., and W.E. Dietrich, Sediment supply, grain size, and rock strength controls on rates of river incision into bedrock, *Geology, 29*, 1087-1090, 2001.

Slingerland, R.L., S.D. Willett, and H.L. Hennessey, A new fluvial bedrock erosion model based on the work-energy principle, *Eos, Trans., Am. Geophys. Union, 78*, F299, 1997.

Small, E.E., R.S. Anderson, and G.S. Hancock, Estimates of the rate of regolith production using Be-10 and Al-26 from an alpine hillslope, *Geomorphology, 27*, 131-150, 1999.

Smith, T.R., B. Birnir, and G.E. Merchant, Towards an elementary theory of drainage basin evolution: I. The theoretical basis, *Comp. and Geosci., 23*, 811-822, 1997a.

Smith, T.R., and F.P. Bretherton, Stability and the conservation of mass in drainage basin evolution, *Water Resourc. Res., 8*, 1506-1529, 1972.

Smith, T.R., G.E. Merchant, and B. Birnir, Towards an elementary theory of drainage basin evolution: II. A computational evaluation, *Comp. and Geosci., 23*, 823-849, 1997b.

Snyder, N.P., K.X. Whipple, G.E. Tucker, and D.J. Merritts, Landscape response to tectonic forcing: Digital elevation model analysis of stream profiles in the Mendocino triple junction region, northern California, *Geol. Soc. Am. Bull., 112*, 1250-1263, 2000.

Stock, J.D., and W.E. Dietrich, Valley incision by debris flows: Field evidence and hypotheses linking flow frequency to morphology, *Eos, Trans. Amer. Geophys. Un., 80*, 473, 1999.

Stock, J.D., and D.R. Montgomery, Geologic constraints on bedrock river incision using the stream power law, *J. Geophys. Res., 104*, 4983-4993, 1999.

Stock, J.D., and W.E. Dietrich, Valley incision by debris flows: evidence of a topographic signature, *Water Res. Res.*, in press.

Strahler, A.N., Equilibrium theory of erosional slopes approached by frequency distribution analysis, *Am. J. Sci., 248*, 673-696, 1950.

Talling, P.J., Self-organization of river networks to threshold states, *Water Resourc. Res., 36*, 1119-1128, 2000.

Tucker, G.E., and R.L. Bras, Hillslope processes, drainage density, and landscape morphology, *Water Resourc. Res., 34*, 2751-2764, 1998.

Tucker, G.E., and R.L. Bras, A stochastic approach to modeling the role of rainfall variability in drainage basin evolution, *Water Resourc. Res.*, *36*, 1953-1964, 2000.

Tucker, G.E., and R.L. Slingerland, Erosional dynamics, flexural isostasy, and long-lived escarpments: a numerical modeling study, *J. Geophys. Res.*, *99*, 12,229-12,243, 1994.

Tucker, G.E., and R.L. Slingerland, Drainage basin responses to climate change, *Water Resourc. Res.*, *33*, 2031-2047, 1997.

van der Beek, P., and J. Braun, Numerical modelling of landscape evolution on geological time-scales: A parameter analysis and comparison with the south-eastern highlands of Australia, *Basin Res.*, *10*, 49-68, 1998.

van der Beek, P., and J. Braun, Controls on post-mid-Cretaceous landscape evolution in the southeastern highlands of Australia: Insights from numerical surface process models, *J. Geophys. Res.*, *104*, 4945-4966, 1999.

Veneziano, D., and J.D. Niemann, Self-similarity and multifractality of fluvial erosion topography 1. Mathematical conditions and physical origin, *Water Resourc. Res.*, *36*, 1923-1936, 2000a.

Veneziano, D., and J.D. Niemann, Self-similarity and multifractality of fluvial erosion topography 2. Scaling properties, *Water Resourc. Res.*, *36*, 1937-1951, 2000b.

Weissel, J.K., and M.A. Seidl, Inland propagation of erosional scarpments and river profile evolution across the Australian southeastern passive continental margin, in *Rivers over Rock: Fluvial processes in bedrock channels*, edited by K. Tinker, and E.E. Wohl, pp. 189-206, American Geophysical Union, Washington, D.C., 1998.

Whipple, K.X., G.S. Hancock, and R.S. Anderson, River incision into bedrock: Mechanics and relative efficacy of plucking, abrasion, and cavitation, *Geol. Soc. Am. Bull.*, *112*, 490-503, 2000a.

Whipple, K.X., N.P. Snyder, and K. Dollenmayer, Rates and processes of bedrock incision by the Upper Ukak River since the 1912 Novarupta ash flow in the Valley of Ten Thousand Smokes, Alaska, *Geology*, *28*, 835-838, 2000b.

Whipple, K.X., and G.E. Tucker, Dynamics of the stream-power river incision model: Implications for height limits of mountain ranges, landscape response timescales, and research needs, *J. Geophys. Res.*, *104*, 17,661-17,674, 1999.

Willett, S.D., Orogeny and orography: The effects of erosion on the structure of mountain belts, *J. Geophys. Res.*, *104*, 28957-28981, 1999.

Willgoose, G. A. Hyspometri curves as an indicator of catchment form: revisited, *Ear. Surf. Proc. and Landforms*, *23*, 611-623, 1998.

Willgoose, G., A statistic for testing the elevation characteristics of landscape simulation models, *J. Geophys. Res.*, *99*, 13,987-13,996, 1994.

Willgoose, G., R.L. Bras, and I. Rodriguez-Iturbe, A coupled channel network growth and hillslope evolution model; 1, Theory, *Water Resourc. Res.*, *27*, 1671-1684, 1991a.

Willgoose, G., R.L. Bras, and I. Rodriguez-Iturbe, A coupled channel network growth and hillslope evolution model; 2, Nondimensionalization and applications, *Water Resourc. Res.*, *27*, 1685-1696, 1991b.

Willgoose, G., R.L. Bras, and I. Rodriguez-Iturbe, A physical explanation of an observed link area-slope relationship, *Water Resourc. Res.*, *27*, 1697-1702, 1991c.

Yatsu, E., On the longitudinal profile of the graded river, *Trans., Am. Geophys. Un.*, *36*, 655-663, 1955.

William E. Dietrich, Department of Earth and Planetary Science, University of California, Berkeley, CA 94720.

Dino G. Bellugi, Department of Earth and Planetary Science, University of California, Berkeley, CA 94720.

Leonard S. Sklar, Department of Earth and Planetary Science, University of California, Berkeley, CA 94720.

Jonathan D. Stock, Department of Earth and Planetary Science, University of California, Berkeley, CA 94720.

Arjun M. Heimsath, Department of Earth Sciences, Dartmouth College, Hanover, NH 03755.

Joshua J. Roering, Department of Geological Sciences, University of Oregon, Eugene, OR 97403.