

# Sensitivity Simulation of Induced Magnetic Fields Associated with Mantle Electrical Conductivity Anomaly

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**Abstract:** Electrical conductivity is sensitive to temperature, partial melt or chemical compositions that could characterize hot plumes. We carried out sensitivity simulations of electromagnetic (EM) induction by the coupling of external EM fields with mantle electrical conductivity anomalies. Results show that the difference of EM responses for different conductivity distributions is observable on the Earth's surface, and that a high conductivity region around the transition zone (i.e., a plume like feature) can be detected from the induced magnetic field above it given an external field in an appropriate frequency band. The different amplitudes as well as the phase shifts of the induced magnetic field are observable between locations relative to the center of the conductivity anomaly. Thus, this simulation study demonstrated that the time-domain code has considerable advantages in dealing with transient EM responses associated with mantle conductivity anomaly.

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## 1. Introduction

Over the past decade seismology has made significant progress in using tomographic techniques for 3-D imaging of the elastic properties in the crust, mantle and core, which dominates our present view of the Earth's interior. Although seismic waves provide good representations of elastic properties, they are not unambiguously sensitive to temperature, partial melt, or chemical compositions within the Earth. In comparison, electrical conductivity is sensitive to such properties and can be measured by studying the frequency-dependent electromagnetic (EM) response in the Earth (Roberts, 1986). However, the distribution of permanent observation sites of EM fields is extremely sparse,

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which has impeded efforts to construct detailed 3-D images of conductivity distribution in the mantle (Schultz and Utada, 1997).

Recent 3-D global seismic tomography studies have revealed low velocity anomalies that are almost continuous from the core mantle boundary (CMB) into upper mantle beneath Africa and the South Pacific (Ritsema *et al.*, 1998; Mégnin and Romanowicz, 2000). The blurred image of a low velocity anomaly having a lateral extent of over 2000 km may indicate the location of a super plume. However, with seismological approaches alone physical characterization of a plume leaves room for debate.

On the other hand, electrical conductivity is sensitive to temperature, partial melt, or chemical compositions (Roberts, 1986; Roberts and Tyburczy, 1999) that could distinguish hot plumes. The lateral temperature contrast across a mantle convection cell is estimated to be in the range of  $10^2$  to  $10^3$  K, and thus the contrast of conductivities between materials with higher temperature and the surrounding mantle may be much larger (by a factor of several or greater) than that observable in seismic properties (within a few percent). This contrast could be effectively used to characterize features associated with hot plumes. Combining electrical conductivity of deep-seated rocks with seismic models would provide a more powerful probe of mantle composition and state than would either property separately.

The present paper reports the results of sensitivity tests of magnetic induction associated with mantle conductivity anomalies using a newly developed time-domain 3-D finite difference code (Chou *et al.*, 2000). This approach is suitable for calculating responses to transient EM fields such as magnetic substorms driven by solar wind, whose predominant frequency band is typically from  $\sim 0.000005$  to  $0.00005$  Hz (i.e., with periods of several hours to 1~2 days). The frequency band translates into a skin depth that ranges from upper mantle to the transition zone.

## 2. Mantle Electrical Conductivity

### 2.1 Layered Models of Conductivity

Traditionally, layered models of electrical conductivity for the mantle have been produced by inverting geomagnetic and magnetotelluric data (e.g., Banks (1969), Barh *et al.* (1993), and Shultz *et al.* (1993)). Such data can be fit to a range of models in which conductivity either varies smoothly with depth or changes abruptly at certain depths (Xu *et al.*, 1998). Due to the spotty distribution of observation stations, and other associated difficulties, no standard mantle conductivity model has been available to date. Recent laboratory experiments measured *in situ* conductivities of mantle minerals under the temperature (T) and pressure (P) conditions in the depth range from upper mantle to  $\sim 1500$  km (Shankland *et al.*, 1993; Xu *et al.*, 1998a, b; Xu and Shankland, 1999), and the conductivity values can be extrapolated to greater depths without gross uncertainty (Shankland *et al.*, 1993; Xu *et al.*, 2000). Figure 1 compares the recent model by Xu *et al.* (1998) with others.

The model by Xu *et al.* (1998) shows a large increase of conductivity in the order of magnitude two at the olivine-wadsleyite transformation at  $\sim 400$  km and a relatively small change at the  $\sim 660$  km discontinuity. The degrees of the discontinuities at these depths are different from those in seismological models (i.e., PREM (Dziewonski and

Anderson, 1981) and *iasp91* (Kennett and Engdahl, 1991)), and models of conductivity distribution derived from observational data (e.g., Schultz *et al.* (1993)).

## 2.2 Conductivity Anomaly in a Hot Plume

Low velocity anomalies that are continuous from the core mantle boundary (CMB) to upper mantle revealed by recent seismic tomography models (Ritsema *et al.*, 1998; Mégnin and Romanowicz, 2000; Obayashi *et al.*, 2001) are primary sources to support the hypothesis of plumes. However, these images are blurred with a lateral extent of over a couple of thousands km and only appear to have a bulk of low velocity anomaly. It is hard to constrain the spatial distribution of the anomalies from these tomography models. Recent numerical modeling that reproduced the volumes and rates of flood basalt eruptions observed at the surface has shown that a plume rising through the mantle in a narrow column, develops a plume head in a thin layer and lets it penetrate rapidly into the base of lithosphere (Leitch and Davies, 2001). This study, however, assumed an eclogite type composition for a plume that can melt at a relatively low temperature. Figure 2 illustrates a hot plume (indicated with shade) upwelling in a narrow, vertical column (tail) and developing an overlying broader layer (head) in a multi-layered mantle structure. It implies the condition that where a plume head is developed is not clear, i.e., below the transition zone, in the transition zone, or in the upper mantle based on available studies.

If a hot plume exists in the mantle, its associated electrical conductivity anomaly can be estimated using *in situ* data measured for various mantle minerals in recent laboratory experiments (Xu *et al.*, 1998, 2000; Xu and Shankland, 1999). To estimate electrical conductivity ( $\sigma$ ) we express it as:

$$\sigma = \sigma_0 \exp\left(-\frac{\Delta H}{kT}\right) \quad (1)$$

where  $\sigma_0$  is a pre-exponential factor,  $T$  is temperature,  $k$  is Boltzmann constant, and activation enthalpy  $\Delta H = \Delta U + P\Delta V$ .  $\Delta U$  is activation energy (eV),  $\Delta V$  is activation volume, and  $P$  is pressure. If the temperature and conductivity in a plume are  $T'$  and  $\sigma'$ , then the contrast of electrical conductivities across the convection cell can be estimated as follows:

$$\ln \frac{\sigma'}{\sigma} = \frac{\Delta H}{k} \left( \frac{1}{T} - \frac{1}{T'} \right) \quad (2)$$

The average temperature in a hot plume is substantially higher than that in the surrounding mantle, and thus  $\sigma'$  is larger than  $\sigma$  in the surrounding mantle.

## 3. Simulation of Electromagnetic Induction

A number of published papers have presented computer simulations for modeling magnetic induction due to the coupling with mantle conductivity distributions. however, most of them were carried out in the frequency domain and not particularly suitable to

model transient responses by 3-D anomalies. Recently Chou *et al.* (2000, 2001) developed 3-D finite difference codes to solve the EM induction equations in the time-domain in both Cartesian and spherical coordinates. The time-domain codes have considerable advantages in dealing with transient EM fields, are designed to run on a high performance computer with parallel processing, and are therefore robust for large-scale modeling incorporating observational data. A brief explanation of the basic equations and coding is given in the Appendix. The present study employs this code in Cartesian coordinates on a regional scale and tests whether EM responses for a suite of conductivity distributions have sufficient resolution and sensitivity for improving mantle structural models associated with a plume.

### 3.1 Time-Domain Finite Difference Computation

Figure 3 illustrates how the grid of the conductivity model is set in Cartesian coordinates: a. top view of the modeling space with a lateral dimension of  $2000 \times 2000 \text{ km}^2$ . The plume head ( $1000 \times 1000 \text{ km}^2$ ) is shaded and accompanied by a tail (shown with dotted lines) beneath it; and b. the grid designed for the finite difference coding. Here the x-axis is in the east-west direction, the y-axis in the north-south direction, and the z-axis in the vertical direction. Depth is in the negative z-direction.

An external EM field (vector potential  $\mathbf{A}$  differentiated by time, i.e., electric field; see eq. (A7) in the Appendix) that oscillates with a period of  $5 \times 10^4$  to  $13 \times 10^4$  sec in x-direction is imposed at the surface. This EM field may sample the transition zone depth according to a skin depth estimate. Periodic boundary conditions are assumed at the four side boundaries. At the bottom boundary (located at 1000km) the vector potential  $\mathbf{A} = 0$  is assumed.

Previously Chou *et al.* (2000) carried out a preliminary sensitivity test of simulation using several layered models of a variety of conductivity values, and showed different responses among the models. They also tested the performance speed and stability/convergence of this scheme on various supercomputers and estimated adequate grid sizes for this simulation. For a region of  $2000 \times 2000 \text{ km}^2$  the computation on Fujitsu VPP700 or similar machines uses a resolution of a grid size of 50 km (x-axis)  $\times$  20 km (y-axis) in lateral extent, and the depth interval of 10 km (z-axis) (Fig. 3(b)). Note that a coarser grid interval is used in the x-coordinate because there is no contribution to the induced field from the derivative in this direction. With this simulation box size ( $2000 \text{ km} \times 2000 \text{ km} \times 1000 \text{ km}$ ), and a time increment of  $dt = 2$  sec, it takes  $\sim 4$  CPU hours with 24 PE's to compute 250000 time steps on the Fujitsu VPP700.

### 3.2 Input Parameters for Modeling

Before computing magnetic induction with a 3-D conductivity anomaly, a standard layered mantle model of conductivity was chosen. As discussed above, however, the modeling is still in a developing stage, and no established standard model is available to date. Differences between models derived from observational data and those from experimental data are also substantial. There are also debates as to where an upwelling plume develops its head (Fig.2). Thus, we test sensitivity of induction for a variety of 3-D conductivity anomalies around the transition zone. The dimension of a vertical column

(plume tail) with conductivity anomaly is varied from  $200 \times 200 \text{ km}^2$  to  $400 \times 400 \text{ km}^2$  embedded in a depth range from 200 km to 1000 km with or without an overlying broader layer ( $\sim 1000 \times 1000 \text{ km}^2$ ) above the column.

Figure 4 shows examples of tested conductivity models. Of the four groups of models, C1 is a 3-layer model consisting of a surface layer ( $\sim$ crust with conductivity  $\sigma_1$ ), a second layer ( $\sigma_2$ ) for upper mantle and a third layer ( $\sigma_3 = (\sigma_4)$ ) for mid mantle (to a depth of 1000 km). Models C2, C3, and C4 have four layers with a lithosphere (to 200 km with  $\sigma_1$ ), upper mantle (200 to 410 km with  $\sigma_2$ ), transition zone (410 to 660 km with  $\sigma_3$ ) and mid mantle (with  $\sigma_4$ ). The conductivity values in the layers of models C2, C3 and C4 are similar to those derived by Xu et al. (1998) (see the caption of Fig. 4 for the conductivity values). The shaded areas indicate upwelling hot materials with conductivity anomalies. The distribution of anomaly is varied in each model. In C1.1, C1.2, C3.1, and C3.2 the plume heads lie beneath the transition zone with tails of a different size in each group. In C2.1, C2.2, and C4.1 the plume heads lie in the transition zone.

In C1, C2 and C3 the anomalies in the shaded areas are five times as large as the surrounding mantle. The anomalies in C4 are estimated for pyrolite or eclogite composition using the temperature difference of  $\sim 500\text{K}$  in eq. (2). In the depth range of 410 to 660 km,  $\Delta H \sim 1.29 \text{ eV}$  for wadsleyite and  $\Delta H \sim 1.16 \text{ eV}$  for ringwoodite. If the temperature contrast is  $T' \sim 2300\text{K}$  vs.  $T \sim 1800\text{K}$ , then,  $\sigma'_3/\sigma_3 \sim 5.7$ . If the composition is more like eclogite (i.e., illuminite and garnet), the activation enthalpy is larger, and  $\sigma'_3/\sigma_3 \sim 10$ . For the depth range of  $\sim 800$  to  $900 \text{ km}$   $\sigma'_4/\sigma_4$  is kept to  $\sim 5$ .

An imposed plane sinusoidal electric field  $\mathbf{E}_x$  (or a vector potential  $\mathbf{A} = (\sin \omega t, 0, 0)$  differentiated by time; see *Appendix*) that oscillates in x-direction with a period of 50000 to 130000 sec ( $\sim 13$  hours to 2 days) represented the external field (Fig. 3b). The induced magnetic field ( $\mathbf{B}_y$  and  $\mathbf{B}_z$ ) on the surface are evaluated after sufficient computation time.

## 4. Results

Throughout this simulation study each run was performed for 3 to 5 times as long as the input oscillation period of the external field to see stable as well as transient responses. Figure 5 shows an example of induction  $\mathbf{B}_y$  as a function of time for 5 cycles for different conductivity models C1.2, C2.2, C3.2 (see Fig. 4). The oscillation period of the input external field is 100,000 sec ( $\sim 28$  hours), and this diagram shows the result for a length of 500,000 sec (5 cycles). The induction was evaluated at the center of the anomaly on the surface (i.e.,  $(x, y, z) = (0, 0, 0)$ ; see Fig. 3b). The induction is stable after the third cycle. Comparative responses differ among the models. Model C2.2 whose plume head lies in the transition zone produced a larger amplitude than models C1.2 and C3.2 with plume heads right below the transition zone.

Figure 6 shows the induction ( $\mathbf{B}_y$ ) for C1.0 and C2.0 that have conductivity anomalies only in a flattened layer, either below or in the transition zone, and no tails. The input period is also 100,000 sec and this diagram is for the first cycle. To evaluate the variation of the induced field among the different observation locations, five other sampling positions (i.e.,  $y=100, 200, 360, 500, 740 \text{ km}$ ) were selected along the  $x = 0$  line (see Fig. 3b). The thick solid lines show the responses at the center of the conductivity anomaly (i.e.,  $(x,y,z)=(0,0,0)$ ), the dotted lines at  $(x,y,z)=(0,740,0)$  that is outside the anomaly, and thinner solid lines at locations inbetween. The response for C2.0 is stronger than

that for C1.0, and can be distinguished among the observation locations in relation to the center of the conductivity anomaly.

Figure 7(a) shows the time evolution of the induced magnetic field  $\mathbf{B}_y$  (in the 5th cycle) for models C1.1, C2.1 and C3.1 whose tail dimensions are larger than those in models C1.2, C2.2 and C3.2. The solid and dotted lines distinguish the locations of induction, i.e., at the center of and outside the anomaly along the  $x = 0$  line as in Figure 6. For C2.1 the difference of induction among the different positions relative to the anomaly center is obvious. The plume head in this model lies in the transition zone. Figure 7(b) shows the induction of  $\mathbf{B}_z$  for C2.1 at different locations (indicated with  $y_1, \dots, \text{and } y_6$ ) along  $x=0$ . The maximum induction is observed right above the conductivity discontinuity ( $y_5=500$  km), and the minimum at the anomaly center. Phase shifts of induction among the different locations are also observable.

Figure 8 shows snapshots of induction (a. for  $\mathbf{B}_y$  and b. for  $\mathbf{B}_z$ ) over the anomaly space at four different times (indicated with short vertical dashed lines in Fig.7). In a. the induction ( $\mathbf{B}_y$ ) is compared between C2.1 (green) and C2.2 (shaded). The column dimensions are  $400 \times 400 \text{ km}^2$  for C2.1 and  $200 \times 200 \text{ km}^2$  for C2.2. Different responses between the models are observable. The induced  $\mathbf{B}_z$  comes from the term  $\partial A_x / \partial y$  (see eq. (A.6)), so it has the maximum amplitude at positions where there is an abrupt change of the conductivity in the  $y$ -direction. In the third layer the conductivity discontinuities are located at  $y = \pm 500$  km, and the absolute amplitudes of  $\mathbf{B}_z$  are maxima along these lines (Fig. 8(b)). The amplitude decreases at locations with increasing distance from these discontinuities.

Figure 9 shows the induction of  $\mathbf{B}_y$  vs. time in the 3rd cycle for C4.1. The snapshots of  $\mathbf{B}_y$  over the plume like anomaly are also plotted in Figure 10 at times indicated with dashed short lines in Fig. 9. The perturbation of induction is observable, and has a sign of the derivative of  $\sim \cos \omega t$ .

## 5. Discussion

Recent studies of seismic waveform modeling demonstrated some resolving power to constrain the finer structures of descending cold slabs, which are revealed as high velocity anomalies in tomographic models (Tajima and Grand, 1998; Tajima *et al.*, 1998). On the other hand, while the substantial low velocity anomalies beneath Africa and south Pacific are primary sources of support for the hypothesis of plumes, physical characterizations using seismological approaches alone are open to a debate.

Large conductivity variations due to the temperature differences across a convection cell can be contrasted with the equivalent P- and S-wave velocity variations, which are in the range of a few percent. Due to the observational limitations, however, EM induction approaches have not yet been utilized to full advantage to improve the Earth's mantle structure. Combining electrical conductivity of deep-seated rocks with seismic models would provide a more powerful probe of mantle composition and state than would either property separately.

We carried out sensitivity simulations of EM responses induced by the coupling of external EM fields with the Earth's mantle, in particular, to see responses due to 3D conductivity anomalies associated with a plume. To make this simulation simple, a plane long-period (50,000 to 100,000 sec) sinusoidal electric field (or a vector potential differen-

ciated by time) oscillating in x-direction was imposed on the surface. Although the setting for the external field is oversimplified, this study demonstrates that magnetic induction has sufficient sensitivity to distinguish conductivity anomalies in the mantle. Results show observable difference of EM responses for different distributions of anomalies, and an anomalous region (i.e., a plume like feature) could be detected given an appropriate frequency band of the external field to sample the depth range. The distribution of anomalies to be detected can be in the depth range from upper mantle, transition zone and mid-mantle to  $\sim 1000$  km.

The penetration of the given electric field into deep mantle should be halted at the anomaly surface. The perturbation of the induction produced over the anomalous region of conductivity (Figs. 8 and 10) is an intriguing implication for this situation. The diffusion time of induction for a medium of conductivity  $\sigma$  with a characteristic depth  $l$  is estimated as

$$T = \frac{l^2}{\eta} \quad (3)$$

(see Jackson (1974))

where

$$\eta = \frac{1}{\sigma \mu_0} \quad (4)$$

$$= 10^7 / 4\pi\sigma$$

If we adopt the conductivity value  $\sigma \sim 0.05$  S/m at a depth immediately above  $\sim 400$  km (at which the surface of the conductivity anomaly is located in C2.1 and C4.1) from Xu *et al.* (1998) and  $l \sim 400$  km, then

$$T \sim 10^4 \text{ sec (or } \sim 2.8 \text{ hour)}$$

In the actual mantle the conductivity values changes with depth, and this estimation is crude. Nevertheless, the simulation results demonstrated that the time-domain code has considerable advantages in dealing with transient EM responses associated with mantle conductivity anomaly. Although we are using a simplified EM field imposed on the surface at present, the codes are flexible and have the ability to incorporate geomagnetic data. Thus, this approach can find applications incorporating transient external EM fields such as magnetic substorms.

The predominant periods of magnetic substorms driven by solar winds are typically several hours to 1~2 days, that translate into a skin depth range from upper mantle to the transition zone (a few hundred to  $\sim 1000$  km). Naturally occurring powerful, low-frequency EM fields whose primary sources are located in the magnetosphere and ionosphere have long been considered to be promising for studying Earth's deep interior in the context of the present study.

The somewhat poorer resolving power of EM imaging techniques (diffusion equation) relative to seismic techniques (wave equation) is counterbalanced by the intense material property contrasts (Shultz *et al.*, 1993) as this study demonstrated. Further development will incorporate simulation codes for external fields and observational data. Although we have here used a simplified EM field imposed on the surface at present, the codes by Choul *et al.* (2000, 2001) are flexible and have capability to incorporate observed data. Results from this kind of simulational study will provide valuable assessments for integration of Earth models.

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## Appendix A. Basic Equations and Skin Depth

The basic formalisms in magnetic induction to represent EM couplings are Maxwell's equations without displacement currents and free of charges:

$$\frac{\partial \mathbf{B}}{\partial t} = -c \nabla \times \mathbf{E}, \quad (\text{A.1})$$

$$\nabla \times \mathbf{B} = \frac{4\pi\sigma}{c} \mathbf{E}, \quad (\text{A.2})$$

$$\nabla \cdot \mathbf{E} = 0, \quad (\text{A.3})$$

$$\nabla \cdot \mathbf{B} = 0, \quad (\text{A.4})$$

where  $\sigma$  is the electrical conductivity and  $c$  is the speed of light. These equations can be simplified by adopting a vector potential  $\mathbf{A}$  and an electrical potential  $\phi$ , with a Coulomb gauge

$$\nabla \cdot \mathbf{A} = 0, \quad (\text{A.5})$$

and the fields are expressed as

$$\begin{aligned} \mathbf{B} &= \nabla \times \mathbf{A}, \quad (\text{A.6}) \\ &= \left( \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}, \frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}, \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right) \end{aligned}$$



$$\mathbf{E} = \frac{-1}{c} \frac{\partial \mathbf{A}}{\partial t} - \nabla \phi, \quad (\text{A.7})$$

If we further assume that there is no static electric field, then Ampere's law Eq. becomes

$$\frac{4\pi\sigma}{c^2} \frac{\partial \mathbf{A}}{\partial t} - \nabla^2 \mathbf{A} = 0. \quad (\text{A.8})$$

This is a diffusion equation.

In the spectral domain the time dependence is written as  $e^{i\omega t}$ , and one can write the solution in 1-D as:

$$A = C e^{z/\lambda + i\pi/4} + D e^{-z/\lambda - i\pi/4}, \quad (\text{A.9})$$

where  $\lambda$  is the skin depth defined by

$$\lambda = \frac{c}{\sqrt{2\pi\sigma\omega}}, \quad (\text{A.10})$$

(Gaussian)

or

$$= \sqrt{\frac{2}{\mu_0\sigma\omega}} \quad (\text{A.11})$$

(M.K.S.)

and  $C, D$  are constants determined by the boundary conditions. The phase  $i\pi/4$  appears because a factor  $\sqrt{i}$  is produced by differentiation. This complex number will produce a phase difference between the time derivative (electric field) and the space derivative (magnetic field) of the vector potential.

In the simulation we solve

$$\sigma \frac{\partial \mathbf{A}}{\partial t} - \nabla^2 \mathbf{A} = 0. \quad (\text{A.12})$$

in which the unit of  $\sigma$  can be expressed as

$$[\sigma_0] = \frac{c^2[T]}{4\pi[L]^2} \text{s}^{-1} = \frac{8.0 \times 10^5[T]}{[L]^2} \text{S/m}. \quad (\text{A.13})$$

We adopt the time unit  $[T]$  as 1 sec, and length unit  $[L]$  as 1 km. In this normalization, the normalization factor is  $[\sigma_0] = 0.8 \text{ S/m}$ . If the conductivities in the Earth interior, that range from  $\sigma = 10^{-3} \text{ S/m}$  on the surface to  $\sigma = 1 \text{ S/m}$  in the transition zone (about 700 km beneath the surface (e.g., similar to model C1 in Fig. 4 and Schultz et al (1993))), then the normalized values range from  $\sigma = 0.00125$  to  $\sigma = 1.25$  in the code.

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