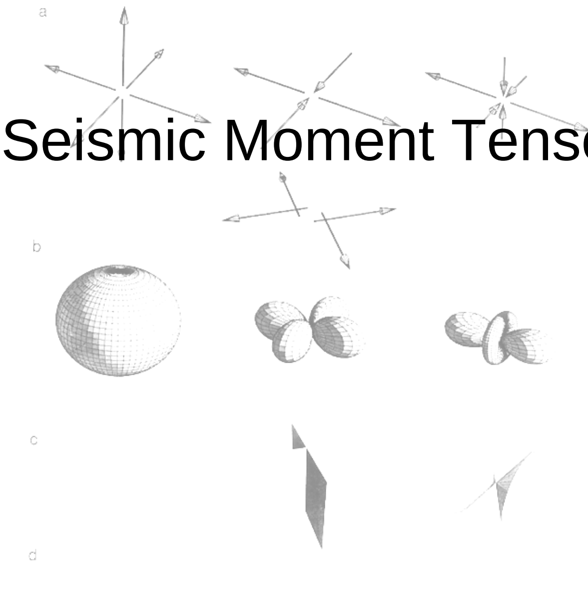
Seismic Moment Tensor



$$U_P(r,t) = \frac{1}{4\pi\rho r\alpha^3} R^P \dot{M}\left(t - \frac{r}{\alpha}\right)$$

$$U_{SV}(r,t) = \frac{1}{4\pi\rho r\beta^3} R^{SV} \dot{M}\left(t - \frac{r}{\beta}\right)$$

$$U_{SH}(r,t) = \frac{1}{4\pi\rho r\beta^3} R^{SH} \dot{M}\left(t - \frac{r}{\beta}\right)$$

What is assumed?

$$U_{SH}(r,t) = \frac{R_{SH}\dot{M}(t - r/\beta)}{4\pi\rho\beta^3 r}$$

Whole Space

$$U_{SH}(r,t) = \frac{2 \cdot R_{SH}\dot{M}(t - r/\beta)}{4\pi\rho\beta^3 r}$$

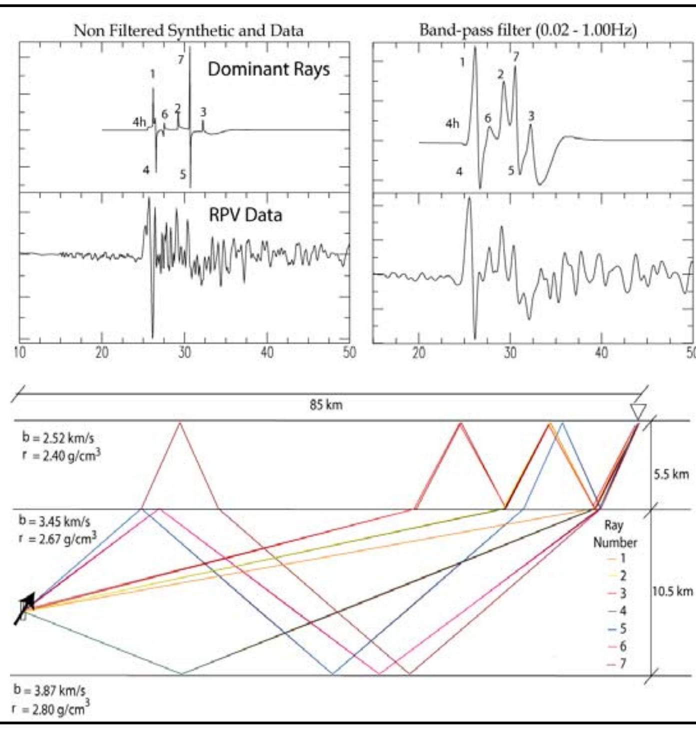
Half Space

$$\frac{2 \cdot \dot{M}(t - r/\beta)}{4\pi\rho\beta^3 r} R_{SH} = U_{SH}(r,t)$$

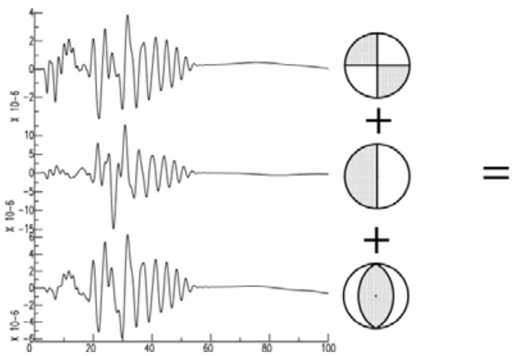
Rewriting

$$Ax = d$$

Linear Equation



Three Fundamental Fault Synthetics



Sum to produce any arbitrary mechanism

$$U_z(r,t) = \sum_{i=1}^{3} A_i \cdot G_i^z(r,t)$$

$$U_r(r,t) = \sum_{i=1}^{3} A_i \cdot G_i^r(r,t)$$

$$U_t(r,t) = \sum_{i=4}^{5} A_i \cdot G_i^t(r,t)$$

$$U_z(r,t) = \sum_{i=1}^{3} A_i \cdot G_i^z(r,t)$$

$$U_r(r,t) = \sum_{i=1}^{3} A_i \cdot G_i^r(r,t)$$

There are 5 independent scaling coefficients (A)

$$U_t(r,t) = \sum_{i=4}^{5} A_i \cdot G_i^t(r,t)$$

The A coefficients are functions of station azimuth, strike, dip and rake.

Approximations of the Representation Theorem

$$u_n(t,\vec{x}) = \int d\tau \iint \left[u_i(\vec{\zeta},\tau)\hat{\nu}_j C_{ijkl}\right] \cdot G_{nk,l}(\vec{x},t-\tau;\vec{\zeta},0)\,d\Sigma$$

$$u_n(t,\vec{x}) = \int \left[u_i(\tau)\hat{\nu}_j C_{ijkl}\Sigma\right] \cdot G_{nk,l}(\vec{x},t-\tau)\,d\tau$$

Spatial point-source

$$u_n(t,\vec{x}) = \left[u_i\hat{\nu}_j C_{ijkl}\Sigma\right] \cdot G_{nk,l}(\vec{x},t)$$

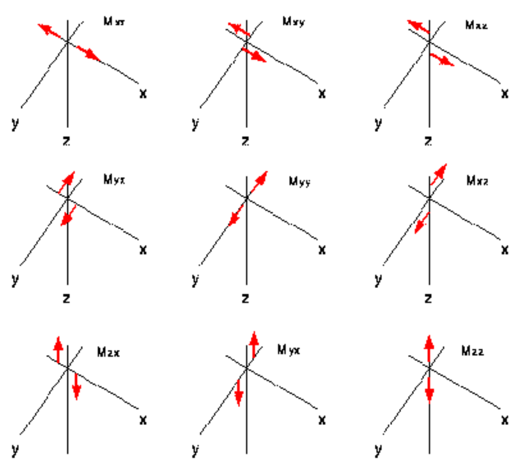
Spatial and temporal point-source

$$u_n(t,\vec{x}) = M_{ij} \cdot G_{ni,j}(\vec{x},t)$$

M has units of moment. i and j refer to directions of forces and derivatives. i.e. they define couples

$$U_n(\vec{x},t) = M_{ij} \cdot G_{ni,j}(\vec{x},t)$$

$$\mathrm{M_{ij}} = \begin{pmatrix} \mathrm{mxx} & \mathrm{mxy} & \mathrm{mxz} \\ \mathrm{mxy} & \mathrm{myy} & \mathrm{myz} \\ \mathrm{mxz} & \mathrm{myz} & \mathrm{mzz} \end{pmatrix}$$



What Kind of Mechanism is this?

$$M_{ij} = \begin{pmatrix} 0 & M_0 & 0 \\ M_0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

What are the principle axes?

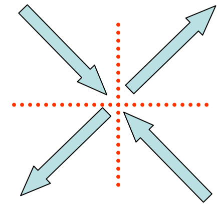
Find Eigenvalues

$$\det \begin{pmatrix} -\lambda & M_0 & 0 \\ M_0 & -\lambda & 0 \\ 0 & 0 & -\lambda \end{pmatrix} = 0$$

$$-\lambda(\lambda^2) - M_0(-\lambda M_0) = \lambda^2 - {M_0}^2 = 0$$

$$\boxed{\lambda = \pm M_0}$$

Find Eigenvectors



$$\begin{pmatrix} -\lambda_i & M_0 & 0 \\ M_0 & -\lambda_i & 0 \\ 0 & 0 & -\lambda_i \end{pmatrix} \vec{a}_i = 0$$

$$\begin{pmatrix} -M_0 & M_0 & 0 \\ M_0 & -M_0 & 0 \\ 0 & 0 & -M_0 \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = 0 \quad \hat{a} = \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0 \right\rangle$$

$$\begin{pmatrix} M_0 & M_0 & 0 \\ M_0 & M_0 & 0 \\ 0 & 0 & M_0 \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = 0 \quad \hat{a} = \left\langle \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0 \right\rangle$$

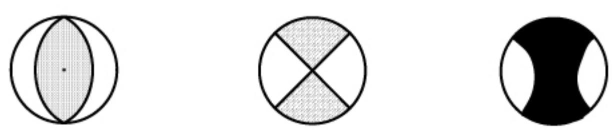
What Kind of Mechanism is this?

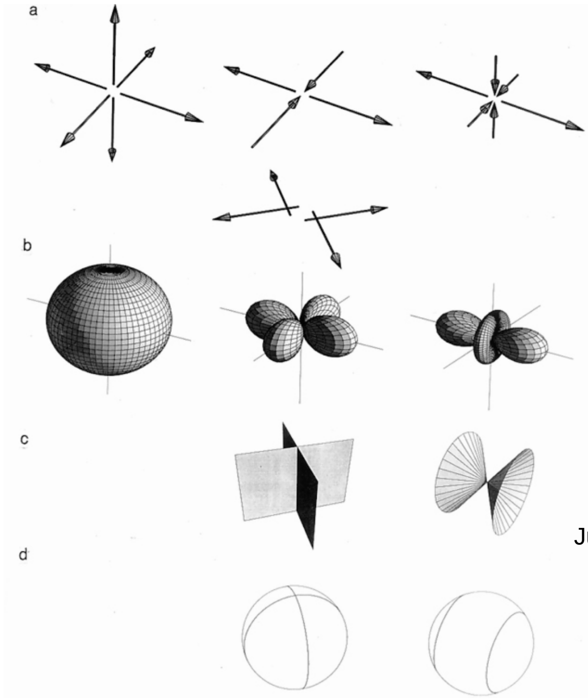
$$M_{ij} = \begin{pmatrix} M_0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -M_0 \end{pmatrix}$$

What are the principle axes?

- Moment tensor is real and symmetric
- It can be diagonalizable to obtain eigenvalues and eigenvectors
- It can be decomposed into fundamental types (double-couple, compensated linear vector dipole, isotropic)
- Decomposition is non-unique (i.e 3DC, 3CLVD, DC+CLVD, Major DC + Minor DC, etc…..)

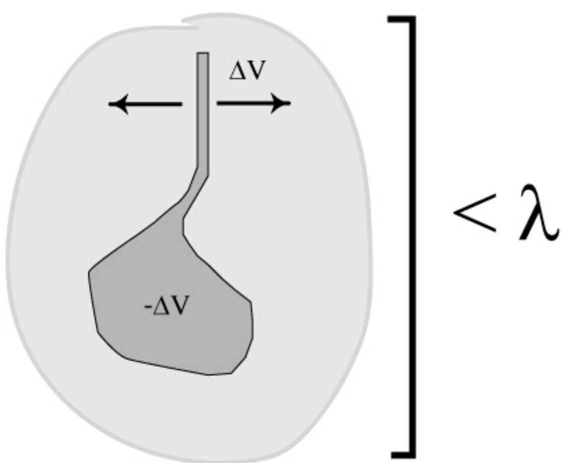
$$\begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} -2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$



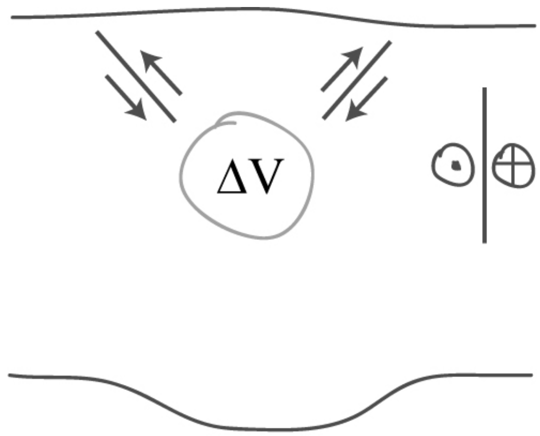


Julian et al., 1998

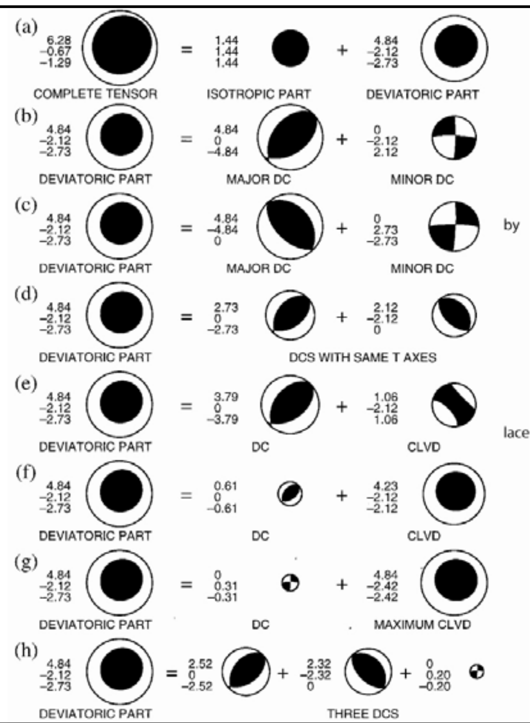
Possible CLVD Mechanism?



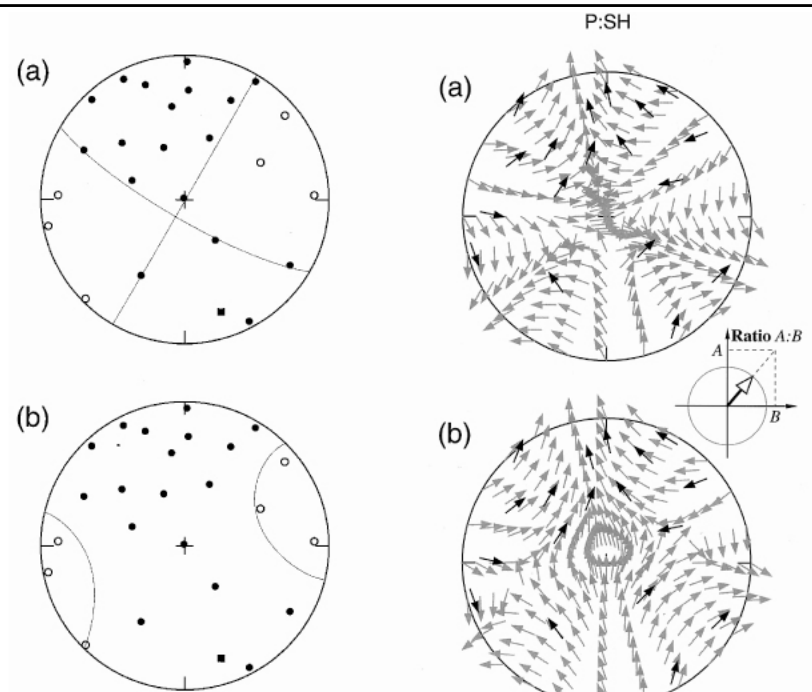
Nuke Mechanism



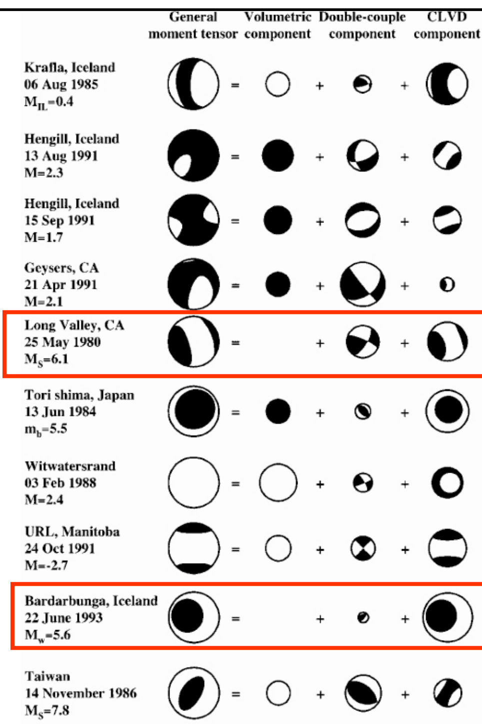
Collapse



Julian et al., 1998



Julian et al., 1998



Miller et al., 1998