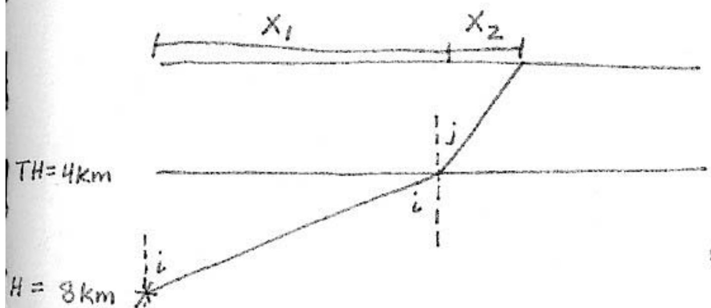
ANSWER SHEET
LAB 4

Exersize 1



$\beta_1 = 2$ km/s
$\rho_1 = 2$ g/cc

$\beta_2 = 3.5$ km/s
$\rho_2 = 2.6$ g/cc

$$\eta_1 = \sqrt{\frac{1}{\beta_1^2} - p^2} = \frac{\cos j}{\beta_1}$$

$$\eta_2 = \sqrt{\frac{1}{\beta_2^2} - p^2} = \frac{\cos i}{\beta_2}$$

$$\mu_1 = \beta_1^2 \rho_1 = 8$$

$$\mu_2 = \beta_2^2 \rho_2 = 31.85$$

A) FOR $X_1 + X_2 = 10$ km

- FIND i

$$X_1 + X_2 = (H - TH)\tan i + TH \cdot \tan j = 10\text{km}$$

$$10\text{km} = (H - TH)\tan i + TH \cdot \tan\left(\sin^{-1}\left(\frac{\beta_1}{\beta_2} \sin i\right)\right)$$

Iterate over i to find the value that yields 10 km.

$i = 62.4°$

- FIND RAY PARAMETER

$$p = \frac{\sin i}{\beta_2} = \frac{\sin 62.4}{3.5 \text{ km/s}} = 0.253 \text{ s/km}$$

- FIND TRAVEL TIME

$$t = p(X_1 + X_2) + (\eta_2)(H - TH) + (\eta_1)(TH) = 4.786 \text{ s}$$

$\eta_1 = 0.431$ s/km
$\eta_2 = 0.132$ s/km

- FIND TRANSMISSION COEFFICIENT

$$T_{21} = \frac{2\mu_2\eta_2}{(\mu_1)(\eta_1) + (\mu_2)(\eta_2)} = 1.1$$

$$\frac{2 \times 31.85 \times 0.132}{8 \times 0.431 + 31.85 \times 0.132} = 1.1$$

Exercise 1

B) For $x_1 + x_2 = 40$ km

- Find i

$$x_1 + x_2 = 40\text{ km} = (H-TH)\tan i + (TH)\tan j$$
$$= (H-TH)\tan i + TH \tan\left(\sin^{-1}\left(\frac{\beta_1}{\beta_2}\sin i\right)\right)$$

$$i = 83.9^\circ$$

- Find Ray Parameter

$$P = \frac{\sin i}{\beta_2} = \frac{\sin(83.7^\circ)}{3.5\text{ km/s}} = 0.284\text{ s/km}$$

- Find Travel Time

$$t = P(x_1 + x_2) + TH\,\eta_1 + (H-TH)\,\eta_2$$
$$= (0.284\text{ s/km})(40\text{ km}) + (4\text{ km})(0.412\text{ s/km}) + (8-4)(0.030\text{ s/km})$$
$$= 13.13\text{ s}$$

$\eta_1 = 0.4115$
$\eta_2 = 0.0305$

- Find Transmission Coefficient

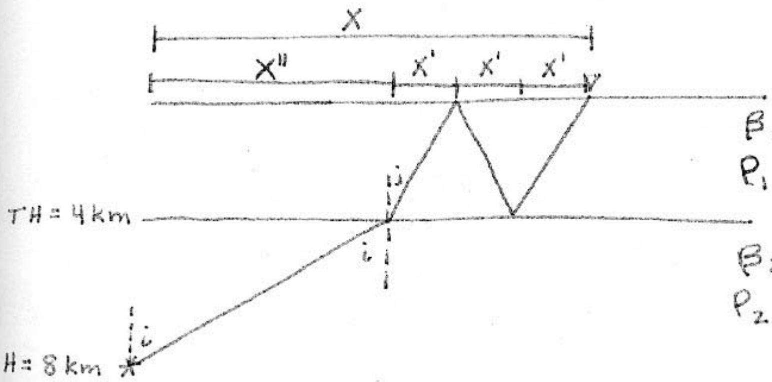
$$T_{21} = \frac{2\mu_2\eta_2}{\mu_1\eta_1 + \mu_2\eta_2}$$

$$= \frac{2(2.6\text{ g/cc})(3.5\text{ km/s})^2(0.030\text{ s/km})}{(2\text{ g/cc})(2\text{ km/s})^2(0.412\text{ s/km}) + (2.6\text{ g/cc})(3.5\text{ km/s})^2(0.030\text{ s/km})}$$

$$= 0.45$$

Thus at grazing incidence the amplitude of the direct arrival is greatly reduced due to energy reflecting downward at the interface. On attenuation curves this would result in greater than 1/r attenuation of the direct phase.

Exersize 2



$\beta_1 = 2$ km/s
$\rho_1 = 2$ g/cc

$\beta_2 = 3.5$ km/s
$\rho_2 = 2$ g/cc

$\eta_1 = \sqrt{\frac{1}{\beta_1^2} - p^2} = \frac{\cos j}{\beta_1}$

$\mu_1 = \beta_1^2 \rho_1$

$\eta_2 = \sqrt{\frac{1}{\beta_2^2} - p^2} = \frac{\cos i}{\beta_2}$

A) For $X = 10$ km

$$10\text{ km} = (H - TH)\tan i + 3TH \tan j$$
$$= (H - TH)\tan i + 3TH\tan\left(\sin^{-1}\left(\frac{\beta_1}{\beta_2}\sin i\right)\right)$$

$i = 47.7°$

- Find Ray Parameter

$$p = \frac{\sin i}{\beta_2} = 0.211 \text{ s/km}$$

- Find Travel Time

$\eta_1 = 0.4531$ s/km
$\eta_2 = 0.1922$ s/km

$$t = pX + \eta_2(H - TH) + 3TH\,\eta_1$$
$$= 8.3 \text{ s}$$

- Find Transmisson Coefficient Total

$$T_{21} = \frac{2\mu_2\eta_2}{\mu_1\eta_1 + \mu_2\eta_2} \qquad R_{10} = 1 \qquad R_{12} = \frac{\mu_1\eta_1 - \mu_2\eta_2}{\mu_1\eta_1 + \mu_2\eta_2}$$

$$\text{Total} = T_{21}\cdot R_{10}\cdot R_{12} = (1.256)(1)(-0.256)$$
$$= -0.322$$

The amplitude is only 1/3 of the amplitude of the direct arrival

Exercise 2

B) FOR $X = 40$ km

$$40\,km = (H-TH)\tan i + 3TH\tan j$$
$$40\,km = (H-TH)\tan i + 3TH\tan\left(\sin^{-1}\left(\frac{\beta_1}{\beta_2}\sin i\right)\right)$$

$$i = 82.8^\circ$$

- FIND RAY PARAMETER

$$P = \frac{\sin i}{\beta_2} = 0.283\ s/km$$

- FIND TRAVEL TIME

$\eta_1 = 0.4119$ s/km
$\eta_2 = 0.0357$ s/km

$$t = PX + \eta_2(H-TH) + 3TH\,\eta_1$$
$$= 16.4s$$

- FIND TOTAL T AND R RESPONSE

$$T_{21} = \frac{2\mu_2\eta_2}{\mu_1\eta_1 + \mu_2\eta_2} \qquad R_{10} = 1 \qquad R_{12} = \frac{\mu_1\eta_1 - \mu_2\eta_2}{\mu_2\eta_2 + \mu_1\eta_1}$$

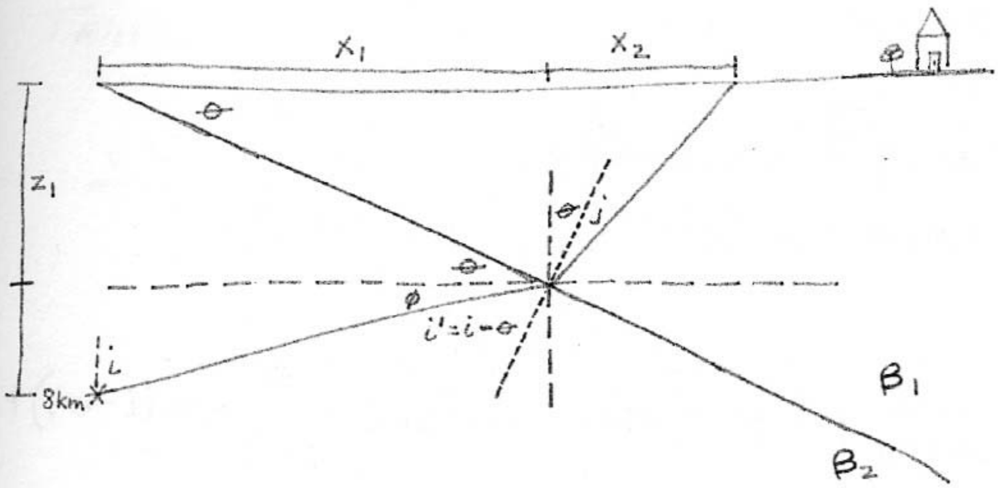
$$TOTAL = T_{21}\cdot R_{10}\cdot R_{12} = (0.513)(1)(0.487)$$
$$= 0.25$$

The multiple is a little more than 1/2 the direct wave amplitude

- $0 \leq T \leq 2$ AND $-1 \leq R \leq 1$

R determines the amplitude and phase
+R → negative amplitude
−R → positive amplitude

Exercise 3



A) FIND $x_1(i, \theta) + x_2(i, \theta) = x_1 + x_2$

$$\theta + \phi + (i - \theta) + 90 = 180$$

$$\phi + i = 90$$

$$\phi = 90 - i$$

$$\tan\theta = \frac{z_1}{x_1} \qquad z_1 = x_1 \tan\theta$$

$$\tan i = \frac{x_1}{(h - z_1)} = \frac{x_1}{(h - x_1\tan\theta)}$$

$$x_1 = h \cdot \tan i - x_1 \tan(i)\tan(\theta)$$

$$x_1 = \frac{h \cdot \tan i}{1 + \tan(i)\tan(\theta)}$$

Exercise 3

$$X_2 = Z_1 \tan(j + \theta)$$

$$X_2 = X_1 \tan\theta \tan(j + \theta)$$

$$X_2 = X_1 \tan\theta \tan\left(\sin^{-1}\left(\frac{\beta_1}{\beta_2}\sin(i-\theta)\right) + \theta\right)$$

$$X_2 = \left(\frac{h \cdot \tan i}{1 + \tan(i)\tan(\theta)}\right)\tan\theta \tan\left(\sin^{-1}\left(\frac{\beta_1}{\beta_2}\sin(i-\theta)\right) + \theta\right)$$

THEREFORE

$$X_1 + X_2 = \frac{h \cdot \tan i}{1 + \tan(i)\tan(\theta)}\left(1 + \tan\theta \tan\left(\sin^{-1}\left(\frac{\beta_1}{\beta_2}\sin(i-\theta)\right) + \theta\right.\right.$$

- ASSUMING $\theta = 5°$ and $X_1 + X_2 = 10$ km, FIND i

  Using the above expression and iterating over i

  $i = 53.0°$

- FIND RAY PARAMETER (10 km)

  $$P = \frac{\sin i}{\beta_2} = 0.228 \text{ s/km}$$

- FIND LOCAL RAY PARAMETER (10 km)

  $$P_L = \frac{\sin(i-\theta)}{\beta_2} = 0.212 \text{ s/km}$$

- FIND TRAVEL TIME (10 km)

  $$t = P(X_1 + X_2) + \eta_2(h - Z_1) + \eta_1^* Z_1 = 3.9$$

  where $\eta_1^* = \sqrt{\frac{1}{\beta_1^2} - P_L^2} = \eta_{1L}$ $\qquad \eta_2 = 0.171$

  $$\sqrt{\frac{1}{\beta_2^2} - P_L^2} = \eta_{2L}$$

Exercise 3

TRAVEL TIME MAY ALSO BE FOUND BY

$$\frac{\sqrt{X_1^2+(h-Z_1)^2}}{\beta_2}+\frac{\sqrt{(X_1+X_2-X_1)^2+Z_1^2}}{\beta_1}=3.9\,s$$

where $Z_1=\frac{h\cdot\tan i}{1+\tan(i)\tan(\theta)}\cdot\tan\theta$

· TRANSMISSON COEFFICIENT (10 km)

$$T_{21}=\frac{2\mu_2\eta_{2L}}{\mu_1\eta_{1L}+\mu_2\eta_{2L}}=1.254$$

$\eta_{1L}=0.453$
$\eta_{2L}=0.191$

increase of 14%.

· FOR $X_1+X_2=40$ km

$i=82.8°$

$P=0.283$ s/km

$P_L=0.279$ s/km

$t=12.9$ s

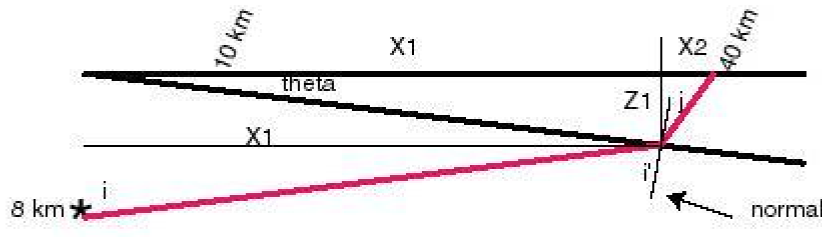
$T_{21}=0.735$ increase of 66% !

$\eta_{1L}=0.415$
$\eta_{2L}=0.06$
$\eta_2=0.035$

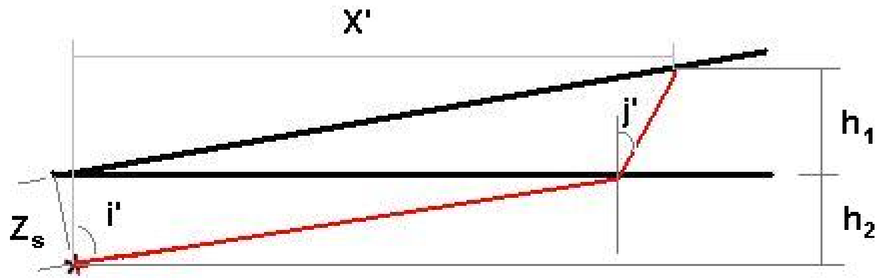
Thus the dipping structure allows for more efficient transmission.

Dipping structure greatly enhances the transmission. Similarly, the R12 reflection can be found to be greater. This type of structure traps energy !

Alternatively, the solution for the ray can be found by rotating the reference system, as shown in the figures bellow:



Rotating the reference system:



$\boldsymbol{i = i' + \theta}$

$$X' = h_1 \cdot \tan\left[\sin^{-1}\left(\beta_1 / \beta_2\right) \cdot \sin(i')\right] + h_2 \cdot \tan(i')$$

$$X' + Z_s \cdot \sin(\vartheta) = (X_1 + X_2) \cdot \cos(\vartheta) \rightarrow \qquad [X - Z_s \cdot \tan(\vartheta)] \cdot \cos(\vartheta) = X'$$

$$Z_s \cdot \cos(\vartheta) = h_2$$

$$X \cdot \sin(\vartheta) = h_1$$

$$\boxed{X_1 + X_2 = \frac{(X_1 + X_2) \cdot \sin(\vartheta) \cdot \tan\left[\sin^{-1}\left(\beta_1 / \beta_2\right) \cdot \sin(i')\right] + Z_s \cdot \cos(\vartheta) \cdot \tan(i')}{\cos(\vartheta)} + Z_s \cdot \tan}$$

For $\theta = 5^o$, $Z_s = 8$ km, $\beta_1 = 2$, $\beta_2 = 3.5$

X = 10 → i' = 48.02^o → i = 53.02^o → X' = 9.26 km

X = 40 → i' = 77.78^o → i = 82.78^o → X' = 39.15 km