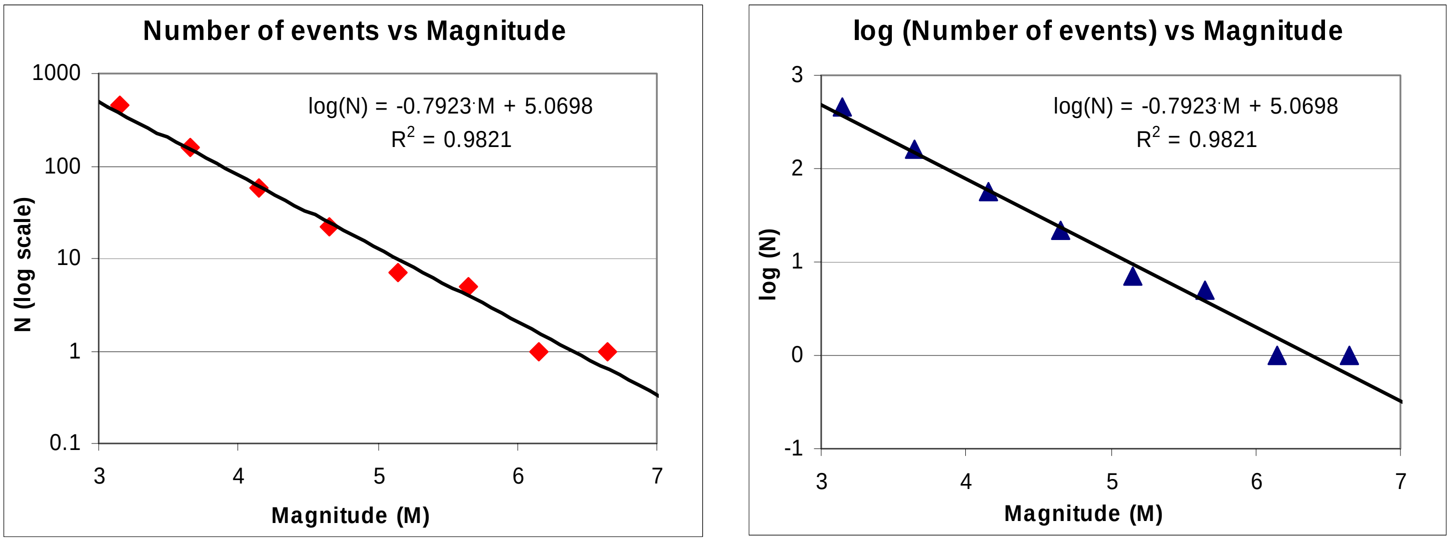
**Exercise 1**: Gutenberg-Richter relationship: $log(N) = a + b \cdot M$

**A1)** For a time period between January 1, 1910 to December 31, 1998

| | 2.9<M<3.4 | 3.4<M<3.9 | 3.9<M<4.4 | 4.4<M<4.9 | 4.9<M<5.4 | 5.4<M<5.9 | 5.9<M<6.4 | 6.4<M<6.9 |
|---|---|---|---|---|---|---|---|---|
| **tot in bin [N] =** | **450** | **161** | **57** | **22** | **7** | **5** | **1** | **1** |
| **Mid Point M** | 3.15 | 3.65 | 4.15 | 4.65 | 5.15 | 5.65 | 6.15 | 6.65 |
| **log (N) =** | **2.65** | **2.21** | **1.76** | **1.34** | **0.85** | **0.70** | **0.00** | **0.00** |




Gutenberg-Richter relationship: $\boldsymbol{log(N) = a + b \cdot M = 5.07 - 0.79 \cdot M}$

For the global scale, typical *b* values are in the range of -2/3 and -1. Therefore this value of b (-0.79) fits well the global data. *b* values describe the proportion of large events to small events. Larger values of *b* indicate a greater proportion of large events.

**A2)** The data is well fitted with this model, especially for small earthquakes (2.9>M>4.9). If a smaller magnitude had been chosen, the data set would have probably been incomplete for this very low magnitude range. In fact, such small earthquakes are not always detected. Furthermore, it is observed that the number of small events in the first 45 years is much smaller than in the following period. This may indicate incomplete dataset due to not very sensitive instrumentation in the first half of the century.

For higher magnitudes (usually those of greatest interest) the model gives a very good average, but it is important to remember that this equation gives the best estimate of what the earthquake will be. It carries uncertainty and therefore, higher and lower magnitudes than the average should be expected. In this case, even if the average is good, the model does not predict the largest event (M = 6.6). From table 1 in the assignment, it can be observed that the "Characteristic Earthquakes" for the Bay Area faults have a recurrence interval greater than 200 years, therefore a database of only 89 years will be incomplete. Part C will discuss more on the time length of the database.

In any case, it is impressive that the rate of occurrence of the earthquakes follow such a simple rule.

**B)**

$$N_{yr} = \frac{N_{89yrs}}{89} = 10^{a-\log(89)+b\cdot M}$$

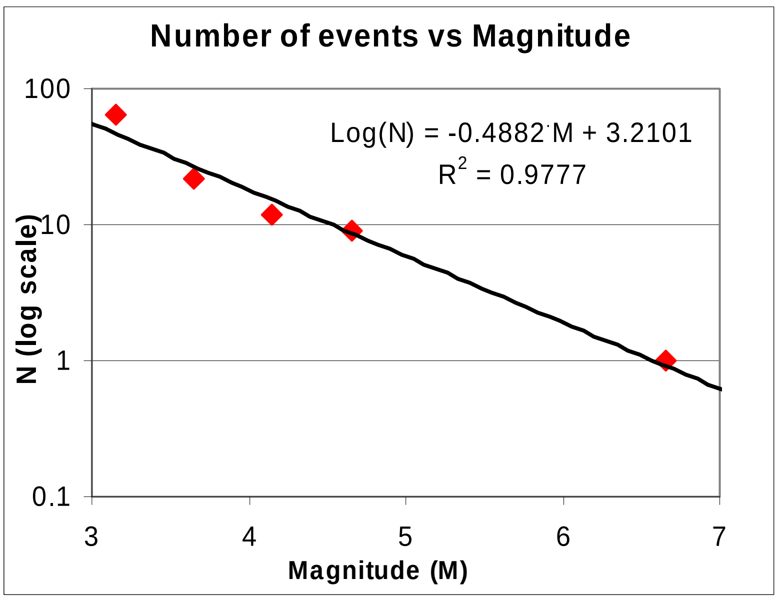
Using the previously estimated parameters (a = 5.07, b = -0.79, for all 89years) we can estimate the magnitude of the 200-year ($N_{yr}$ = *1/200*) event:

**The magnitude of a 200-year event is M = 6.9**

Compared to the size and recurrence of the events forecasted to occur on the Bay Area Faults, the prediction seems very accurate.

**C)** First half-period (1910-1954)

| | 2.9<M<3.4 | 3.4<M<3.9 | 3.9<M<4.4 | 4.4<M<4.9 | 4.9<M<5.4 | 5.4<M<5.9 | 5.9<M<6.4 | 6.4<M<6.9 |
|---|---|---|---|---|---|---|---|---|
| **tot in bin [N] =** | **65** | **22** | **12** | **9** | **-** | **-** | **-** | **1** |
| **Mid Point M** | 3.15 | 3.65 | 4.15 | 4.65 | 5.15 | 5.65 | 6.15 | 6.65 |
| **log (N) =** | **1.81** | **1.34** | **1.08** | **0.95** | **-** | **-** | **-** | **0.00** |




Based only on this 45 years:

$$N_{yr} = \frac{N_{45yrs}}{45} = 10^{a-\log(45)+b\cdot M}$$

And using the previously fitted parameters:

*a = 3.21* *b = -0.49*

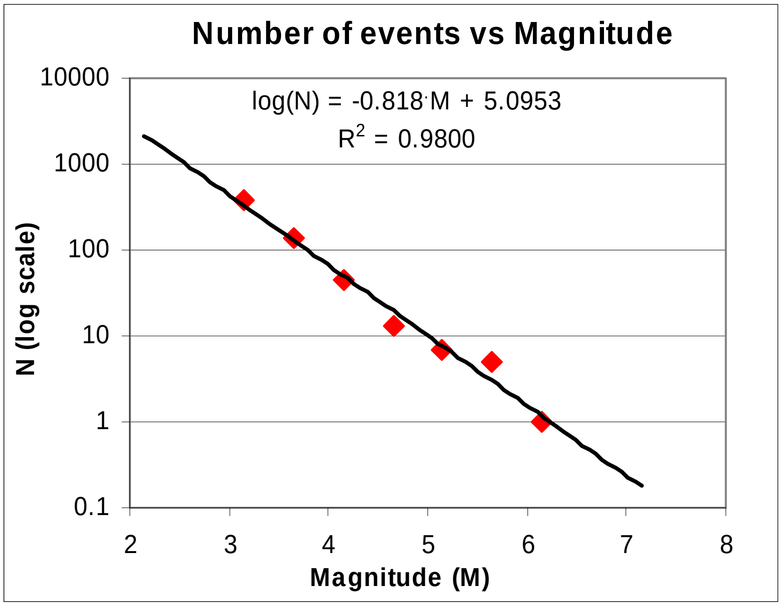
***log(N) = a + b·M =3.21 – 0.49·M***

we can estimate the magnitude of the 200-year ($N_{yr}$ = *1/200*) event:

**The magnitude of a 200-year event is M = 7.9**

Second half-period (1955-1998)

| | 2.9<M<3.4 | 3.4<M<3.9 | 3.9<M<4.4 | 4.4<M<4.9 | 4.9<M<5.4 | 5.4<M<5.9 | 5.9<M<6.4 | 6.4<M<6.9 |
|---|---|---|---|---|---|---|---|---|
| **tot in bin [N] =** | **385** | **139** | **45** | **13** | **7** | **5** | **1** | **0** |
| **Mid Point M** | 3.15 | 3.65 | 4.15 | 4.65 | 5.15 | 5.65 | 6.15 | 6.65 |
| **log (N) =** | **2.59** | **2.14** | **1.65** | **1.11** | **0.85** | **0.70** | **0.00** | **-** |




Based only on this 44 years:

$$N_{yr} = \frac{N_{44yrs}}{44} = 10^{a-\log(44)+b\cdot M}$$

And using the previously fitted parameters:

$a = 5.10$ $b = -0.82$

$$\boldsymbol{\log(N) = a + b \cdot M = 5.10 - 0.82 \cdot M}$$

we can estimate the magnitude of the 200-year ($N_{yr} = 1/200$) event:

**The magnitude of a 200-year event is M = 7.0**

In probabilistic analysis, the amount of data often influences the result of the analyses. When, increasing the amount of information does not change the results, it is said that the model arrives to convergence.

In this case the models have not yet arrived to convergence. It is demonstrated by the differences in the parameters that best fit the Gutenberg-Richter equation.

The following are the results of the fit in the given time windows:

1910-1998: $\ln(N) = -0.79 \cdot M + 5.07 \Rightarrow M_{200yr} = 6.9$

1910-1954: $\ln(N) = -0.49 \cdot M + 3.21 \Rightarrow M_{200yr} = 7.9$

1955-1998: $\ln(N) = -0.82 \cdot M + 5.10 \Rightarrow M_{200yr} = 7.0$

The difference in the estimates is not negligible.

The greater prediction given by the first time window (1910-1954) is due to the fact that a large magnitude event was observed in this time window. This large M was observed in a period of only 45 years, while when considering the complete data (1910-1989) the same event was seen once in 89 years.

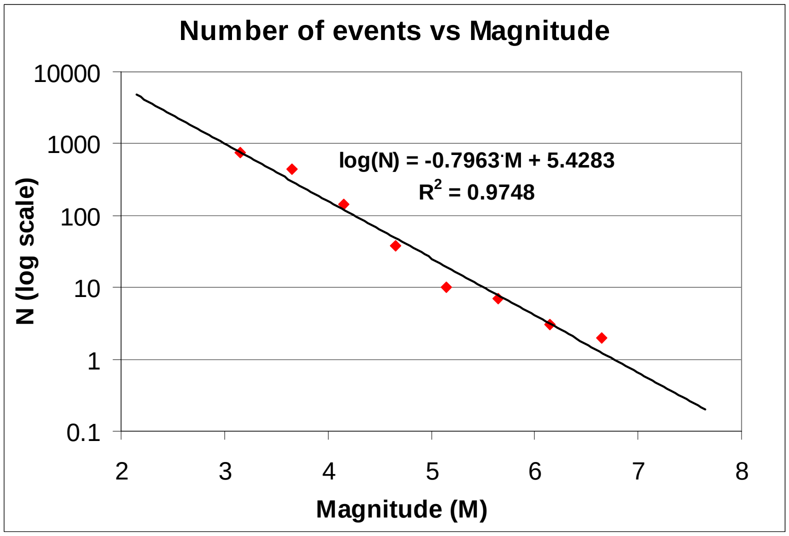
Several of you wrote that the *a* value represents the maximum earthquake magnitude expected over the studied period. If the Gutenberg-Richter relationship is solved for *M:* $M = \frac{\log(N) - a}{b}$

Then, for *N = 1* (largest event expected on the studied period): $M = -a/b$, therefore this ratio is the largest expected event, not only the *a* value. In many cases, the value of *b* is equal to -1, and therefore, in those cases, *a* does represent the largest event expected on the studied period.

For this case: $M = -a/b = -5.07/-0.79 = 6.42$, which as described earlier, under-predicts the largest event, but not by much.

**Exercise 2**: Mendocino

| | 2.9<M<3.4 | 3.4<M<3.9 | 3.9<M<4.4 | 4.4<M<4.9 | 4.9<M<5.4 | 5.4<M<5.9 | 5.9<M<6.4 | 6.4<M<6.9 |
|---|---|---|---|---|---|---|---|---|
| tot in bin [N] = | 764 | 444 | 144 | 38 | 10 | 7 | 3 | 2 |
| Mid Point M = | 3.15 | 3.65 | 4.15 | 4.65 | 5.15 | 5.65 | 6.15 | 6.65 |
| log (N) = | 2.88 | 2.65 | 2.16 | 1.58 | 1.00 | 0.85 | 0.48 | 0.30 |




Based only on this 44 years:

$$N_{yr} = \frac{N_{44yrs}}{44} = 10^{a-\log(44)+b\cdot M}$$

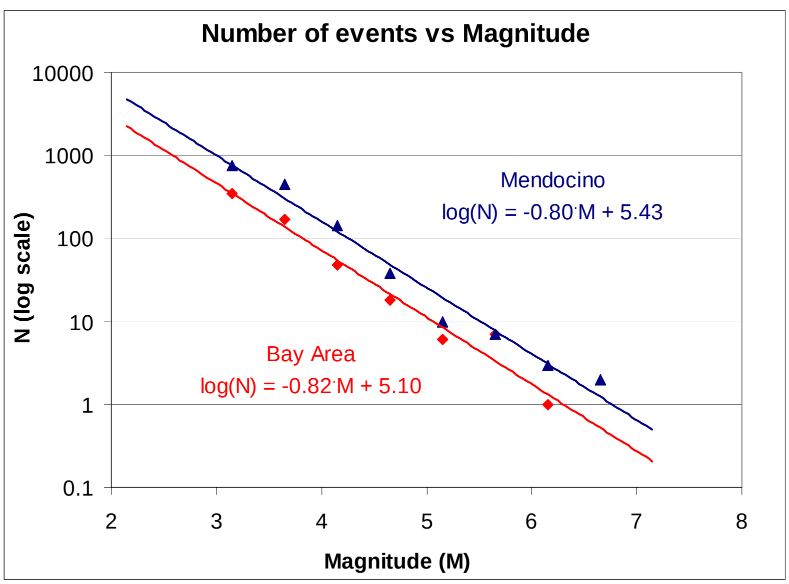
And using the previously fitted parameters:
*a = 5.43* *b = -0.80*

$$\boldsymbol{\log(N) = a + b\cdot M = 5.43 - 0.80\cdot M}$$

we can estimate the magnitude of the 200-year ($N_{yr} = 1/200$) event:

**The magnitude of a 200-year event is M = 7.6**

The similar *b* values in both the Bay Area and Mendocino indicate that the proportion of large events to small events in both areas is similar. Given similar *b* values, the greater *a* value in Mendocino indicates that this is a more active region.




The plot shows the best fit to the seismicity data recorded in Mendocino and Bay Area in the period 1955-1989.

They both have pretty similar *b* parameter, and therefore almost parallel curves. The main difference is in the *a* value, where the Mendocino data shows a larger intercept, and therefore a more active zone zone.

In fact, the predicted 200 year event based on this time window is larger for the Mendocino area than for the Bay Area:

Mendocino: $\Rightarrow M_{200yr} = 7.6$

Bay Area: $\Rightarrow M_{200yr} = 7.0$

Since the *b* values for both time periods are similar: $\frac{N_{yr-Mendocino}}{N_{yr-Bay\ Area}} = \frac{(10^a \cdot 10^{b\cdot M})_M}{(10^a \cdot 10^{b\cdot M})_{B\ A}} \approx \frac{(10^a)_M}{(10^a)_{B\ A}} = 10^{0.28} \approx 2$

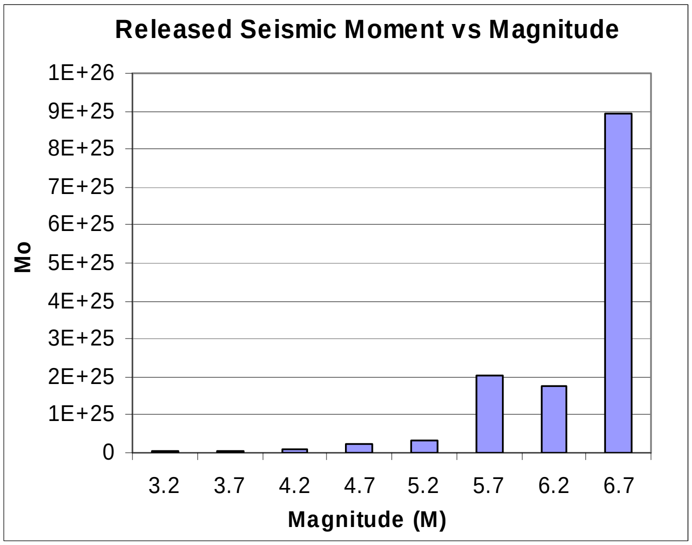
Therefore, the Mendocino area is about "twice as active" as the Bay Area

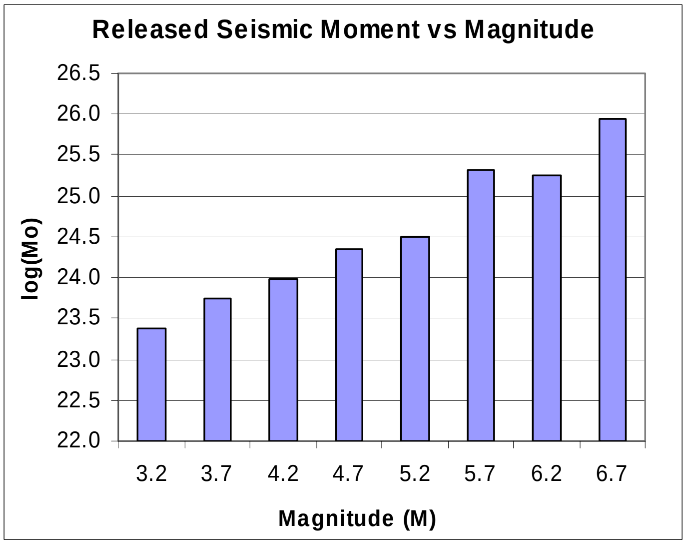
**Exercise 3**:

| | 2.9<M<3.4 | 3.4<M<3.9 | 3.9<M<4.4 | 4.4<M<4.9 | 4.9<M<5.4 | 5.4<M<5.9 | 5.9<M<6.4 | 6.4<M<6.9 |
|---|---|---|---|---|---|---|---|---|
| **tot in bin [Mo] =** | **2.444E+23** | **5.45001E+23** | **9.796E+23** | **2.21E+24** | **3.186E+24** | **2.037E+25** | **1.758E+25** | **8.913E+25** |
| | 3.15 | 3.65 | 4.15 | 4.65 | 5.15 | 5.65 | 6.15 | 6.65 |
| **log (Mo) =** | **23.39** | **23.74** | **23.99** | **24.34** | **24.50** | **25.31** | **25.25** | **25.95** |

I made this calculation using the moment released for each event. You may have found it easier to use the table in exercise 1 and applying the formula: $Mo = N \cdot 10^{1.5M + 16.05}$

If you used this second (approximation) approach, your numbers will be slightly different from mines.



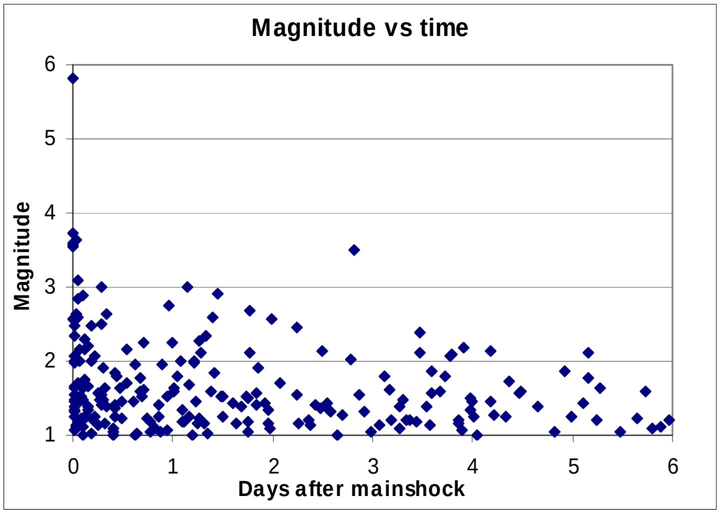

Fraction of total Mo over 89 years that the M6.6 event released:

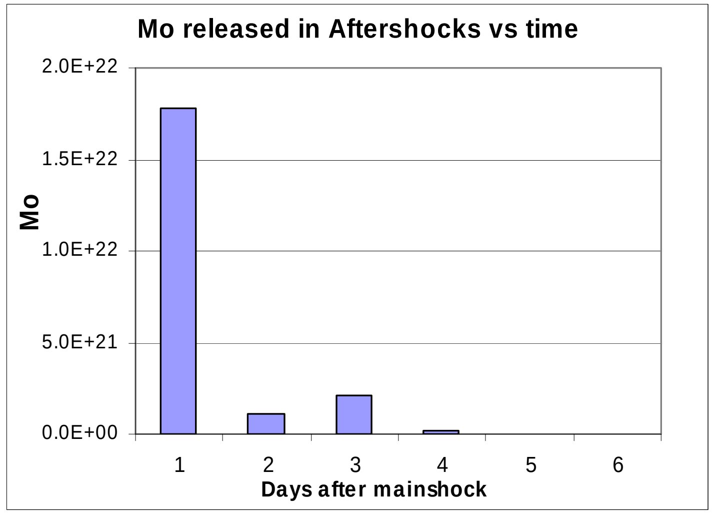
$$\frac{Mo_{M\,6.6}}{Mo_{tot}} = \frac{8.913 \cdot 10^{25}}{1.342 \cdot 10^{26}} = 66.4\%$$

Or using the approximation described above:

$$\frac{Mo_{M\,6.6}}{Mo_{tot}} = \frac{1.06 \cdot 10^{26}}{1.50 \cdot 10^{26}} = 71\%$$

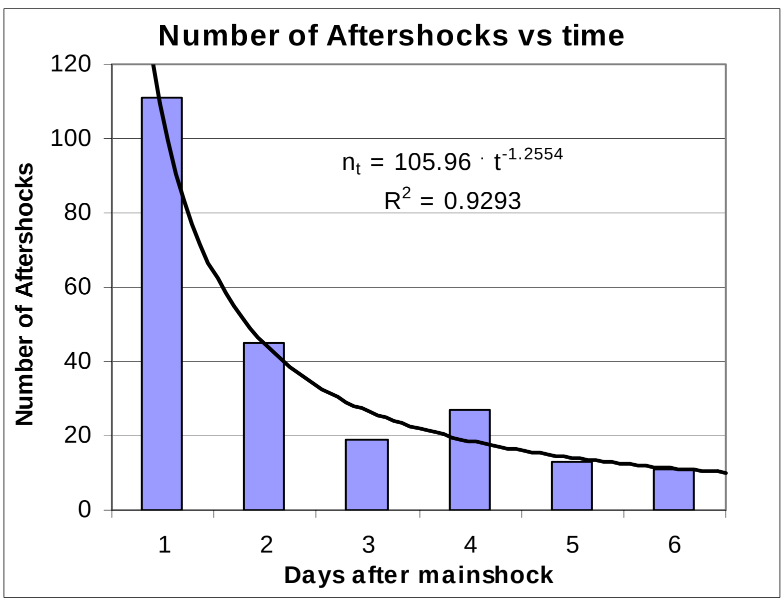
**Exercise 4**: **A)**




The left plot shows a high concentration of aftershocks in the close time frame after the mainshock. This can be noticed by the high concentration of events in the first 24 hours after the mainshock. Ignoring the M3.5 event at the end of day 2, a pretty clear trend in the reduction of the maximum magnitude of the aftershocks with time can be noticed. This trend becomes even clearer if we plot the moment magnitude released by the aftershocks in 24hour bins, as shown in the right plot.

**B)** Omori's equation fits reasonably well the data of from the Morgan Hill's earthquake. In this case the equation is given by (assuming K = 0):




$$n_t = \frac{C}{(K+t)^P} = \frac{106}{t^{1.255}}$$

This result is within the usual observed values for the decay rate (1.0 < P < 1.4).

As shown in the plot, the data is well fitted by the Omori's Law. The data appears to have inverted days 3 and day 4 in the frequency of earthquakes. This feature can be explained by observing the data in the Magnitude vs. Time plot. On day 3 a relatively high aftershock was observed, concentrating energy dissipation on it, and at the same time, the events on day 4 could be interpreted as aftershocks of this later event *plus* aftershocks of the main event.

**C)** Based on Omori's equation, and assuming that the background rate of M>1.0 events before the earthquake was 1 per week, the time it would take before the rate of earthquake occurrence in the area returns to the background rate can be computed as:

$$t = \left(\frac{C}{n_t}\right)^{1/P} = \left(\frac{106}{1/7}\right)^{1/1.255} \approx 193 \;\; days \approx 6.5 \;\; months$$

This Law predicts a very long time for the seismicity to go to pre-event rates.